AC INSTRUMENTATION
APPLICATION NOTES

1973

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The Singer Co., Los Angeles Operation
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APPLICATION NOTE NO. 1
SYNCHRO AND RESOLVER TESTING

by

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***Director of Engineering, Los Angeles Operation**

Synchro and Resolver Standards

Prior to the introduction of synchro and resolver standards, "master" transmitters were simulated with an ordinary synchro or resolver transmitter together with an angular positioning device, a synchro or resolver bridge, a phase-sensitive null detector, a bridging transformer, and a phase shifter. This technique required the operator to "calibrate" the transmitter against the bridge at *each* angle by adjusting the angular positioning device for a null on the detector. The purpose of the phase shifter, required by both military and commercial specifications, was to compensate for the time phase shift of the transmitter. The main disadvantages of this technique are that it is relatively slow and inaccurate, besides being difficult to operate, expensive, and quite space consuming. The inaccuracy of this method is due to several causes, such as operator error, drift, hookup error, plus the specified error of the individual pieces of equipment.

The requirement for an efficient and accurate "master" synchro and resolver transmitter was met by a simple extension of the time-proven *RatioTran* technique, pioneered by *Gertsch* over a decade ago. This device, which replaces all of the equipment required by the earlier technique, was soon termed a synchro or resolver standard. Basically, a *Gertsch* synchro or resolver standard consists of an extremely low phase shift transformer with a primary and one or two *isolated* secondaries respectively. The secondary has several precision taps which determine the angles available. Depending upon the particular model, angles may be selected by several techniques such as by turning one or more knobs, pulsing a rotary solenoid operated switch, or by energizing a relay.

Besides possessing a high degree of no-load accuracy (2 seconds-of-arc traceable to NBS) standards generally have a very low nominal output impedance, and an extremely low output impedance unbalance. This allows a normal load to be applied without significantly affecting the no-load accuracy. The output impedance of synchro and resolver standards is typically a few ohms; however, specific values of output impedance e.g., 50 + j50 ohms can be provided for applications where it may be desirable to actually simulate the output impedance of a synchro or resolver. In order to prevent angular errors due to loading from occurring, it is necessary for the output impedance of the standard to be balanced at each angular position. Output impedance unbalances as low as .01 + j.001 ohms are considered standard.

In addition to maintaining a high degree of accuracy under load, the standard may normally be operated over a wide voltage (10:1), and frequency range with very little degradation of accuracy. The frequency range of synchro and resolver standards extends from 30 cps to 10 KC or higher. Ideally, units are designed for a particular frequency; however, the useful frequency range for a particular unit generally extends to approximately two octaves (four times) above the design frequency. Operation below the design frequency will saturate the transformer, unless the input voltage is reduced in direct proportion to the frequency.

Normally, high accuracy resolver standard applications require that the outputs of the standard be isolated from each other. If the sine and cosine outputs of the standard are common, an unsymmetrical current will be forced to flow through the interwinding capacitance of the resolver under test; therefore, causing angular errors to occur which under actual conditions do not exist.

Synchro and Resolver Bridges

Basically there are two types of synchro and resolver bridges – the resistive (10K) and the inductive. Up until a few years ago, the resistive-type bridges were the only type of bridges available; however, the demand for higher input impedances, lower output impedances, and higher accuracies brought about another extension of the *RatioTran* technique – the inductive-type bridge. Actually, the term inductive-type is somewhat misleading, since the inductive-type bridge is normally operated in the vicinity of self-resonance. Therefore, the input impedance may be slightly capacitive, resistive, or slightly inductive. Obviously, the highest input impedance will be obtained at the resonant frequency.

Compared to resistive-type bridges, inductive-type bridges have an input impedance almost two orders of magnitude greater, and an output impedance approximately three orders of magnitude lower. The inherently high output impedance of the resistive-type bridge severely limits its application at high frequencies, since internal and external stray capacitances are more likely to alter the tapped ratios and the critical impedance balance of the four 10K resistors.

It is a fact that the input impedance of an inductive-type bridge is somewhat a function of voltage; therefore, it is difficult to maintain a real high degree of input impedance balance. However, it should be noted that the higher the input impedance, the less the effect of input impedance unbalance. In addition, $1/\sin 2\theta$ variations in input impedance unbalance are allowable without increasing the angular error.

Both the resistive and the inductive types of bridges are available either compensated or uncompensated. A compensated bridge has the same error voltage gradient at each angular position. Resistive-type bridges are compensated by using different resistive dividers at each angle, while the inductive-type are compensated with the use of a scaling transformer. The output impedance of the compensated resistive-type bridge is approximately four times higher than the uncompensated type.

An often overlooked accessory piece of equipment required for synchro and resolver bridges is a bridging transformer, when using a single-ended null detector. Grounding one terminal of the bridge output directly can cause apparent angular errors to occur. All *Gertsch* compensated bridges utilize a combination scaling and double-shielded bridging transformer. If a compensated resistive-type bridge is used, a bridging transformer with an input impedance of a megohm or more is required, otherwise the compensation will be affected due to loading. In addition, it may be necessary to use a bridging transformer which has been resonated, in order to prevent phase shift of the error voltage from the output of the resistive-type bridge.

Gertsch uses a unique resolver bridge circuit which provides two isolated and double-shielded inputs, plus an isolated and double-shielded output. In addition, the circuit employed is inherently compensated. Resistive-type 4-arm resolver bridges effectively interconnect the center taps of the two output windings of the resolver under test through a 2.5K ohm resistance which has no effect *if* the outputs of the resolver under test are balanced to each other.

Inductive-type bridges are ideally designed for a particular frequency; however, the useful frequency range generally extends from one octave below (one-half) to one octave above (twice) the design frequency. The frequency range of inductive-type bridges is from 30 cps to 10 KC, or higher. At frequencies below 1600 cps, input impedances as high as 800K are available. The input impedance obtainable at 10 KC is approximately 250K.

Another often-heard argument against resistive-type bridges is that 10K ohms is not a realistic load. In fact, the 10K ohm load can cause angular errors to occur which under actual conditions would never exist, since the load is normally quite inductive. This is particularly true when testing synchros and resolvers with a high output impedance. Testing under load is normally accomplished by simply connecting the actual load, e.g. a CT, in parallel with the input to the bridge. Therefore, if the load impedance is a few thousand ohms or higher the input impedance of the resistive-type bridge will alter the apparent load impedance seen by the component under test. The input impedance of the inductive-type bridge, however, can normally be neglected.

Ratio accuracies equivalent to 2 seconds-of-arc are easily obtained and maintained with inductive-type bridges, while 2 seconds-of-arc is quite difficult to maintain with resistive-type bridges due to aging, temperature coefficients, and accidental overloading. Overloading of a resistive-type bridge can be caused, for example, by accidentally shorting one of the stator input leads to one of the output terminals. Although an overload may not be sufficient to burn-out the bridge, it is quite possible to permanently change the tap ratios and the total resistance values of the four arms. A damaged bridge could go undetected for several months, until it's recalled for periodic "calibration."

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First Edition 5-24-63, Second Edition 5-17-72

Whichever type of resolver bridge is used, resistive or inductive, the user should be certain not to use the type of bridge which commons the sine and cosine outputs of the resolver under test, unless large angular errors can be tolerated. Such errors are a function of the source impedance and the interwinding capacitance of the resolver under test, neither of which are generally known by the bridge user. Therefore, it is best to avoid this type of bridge entirely.

High Resolution Standards and Bridges

Conventional 5° resolution standards and bridges are generally inadequate for servo system or module testing, where convenient values of system parameters (knots, yards, etc.) are not even multiples of 5°. In general, high-resolution (.001°) decade standards and bridges are used: (1) As a distinctly superior system-error bridge, for measuring the output angle of synchros or resolvers which are an integral part of a system or sub-system; (2) for trouble-shooting sources of error in data transmission systems such as cable loading, gearing, dial readout, impedance unbalance, etc.; (3) to generate precise input angles to a system or sub-system; (4) to measure system sensitivity or response to small angular errors; (5) for measuring the drift or stability of a system about a particular point.

The theory of operation of these *Gertsch* instruments is basically the same as the 5° units, except that several precision transformers and switches have been added to effectively multiply the number of taps by 5,000 in order to provide .001° resolution.

A very important feature of the *Gertsch* decade bridges is their complete independence of synchro or resolver functional accuracy, transformation ratio, and phase shift, plus their completely passive circuitry. Absolutely no preliminary calibration manipulations are required.

Functional Accuracy Testing

The angular accuracy of a data-transmission synchro or resolver is based on the *ratio* of two output voltages to each other. The functional accuracy of a computational resolver, however, is based on the ability of each output winding, individually, to produce a voltage proportional to the sine or cosine of the shaft angle. The output 90° from minimum coupling is normally used as the reference voltage.

The procedure for measuring functional accuracy consists of setting up a voltage across a voltage divider which is equal in amplitude and phase to the output of the winding under test at 90° from minimum coupling. The output of the resolver at each angle is then bridged against the output of the voltage divider. A computational resolver with a functional accuracy of 0.05% would be allowed ± 500 ppm deviation from the theoretical sine or cosine value at each angle. Note, that the allowable *angular* error is a function of the angular position of the shaft. For example, a 0.05% functional error corresponds to an angular error of 1.7 minutes at 0°, and 3.4 minutes at 60°.

In practice, the above technique may be instrumented in either one of two ways. The first technique employs a Sine θ *RatioTran* (sine 0° to 90° in 5° steps), a Phase Angle Voltmeter (PAV), a conventional *RatioTran*, and an RC phase shifter. The functional error at each angular position is displayed on the PAV as an error voltage. For example, a 0.05% computational resolver with a maximum output of 15V, is allowed ±7.5 mv at *each* angular position. With the variable range attenuator on the PAV, 7.5 mv can be set up to read full-scale on the 3 mv range. Therefore, the functional error at each angle can be read directly on the PAV meter.

The second technique utilizes a *Gertsch* Complex Ratio Bridge (Model CRB-2B) and a conventional *RatioTran*. The *RatioTran* is adjusted for a null on the CRB with the resolver set to 90° from minimum coupling. The inphase *RatioTran* on the CRB is adjusted for a null at each angle, and its deviation from the theoretical sine or cosine of the shaft angle is noted in ppm. The quadrature *RatioTran* at null, can be set up to indicate the phase shift from primary to secondary, directly in degrees.

Automatic Testing

Gertsch developed two lines of automatic synchro/resolver standards and bridges, the rotary solenoid type and the relay (programmable) type. Basically the rotary solenoid type is ideal for component angular accuracy tests, since the unit sequentially advances in 5° steps with each contact closure. On the other hand, the relay type is ideal for system testing where random access to several angles may be desired. Input commands can be in several forms such as BCD, straight binary, decimal, etc.

To accompany the rotary solenoid type units in automatic component tests, *Gertsch* developed an automatic dividing head. This automatic unit advances 5° every six seconds, and remains there for approximately 4 seconds. In addition to automatically advancing itself, it also advances the rotary solenoid standard or bridge, mutes the phase angle voltmeter, and lowers the pen on the DC recorder at the proper moment. The timing sequence for this remote control is programmed by cam-operated switches. The camshaft is driven from the same synchronous motor that drives the 5° indexing mechanism.

With the automatic test set-up above, it is possible to complete a 72 point angular accuracy test in less than 10 minutes including set-up time. If a strip chart recorder is used, a permanent record of the components error can also be made automatically. Manual techniques can take as long as an hour or more — even with an experienced operator.

Dividing Heads

Unlike most other test instruments used in angular accuracy tests, not all dividing head errors can be measured, or specified. The most common specification for dividing heads is the indexing accuracy. Indexing accuracies currently available range from 20 seconds-of-arc down to ¼ of a second-of-arc. Additional errors depend upon the way in which the component under test is mounted in the dividing head, and the environmental conditions. An ideal mounting technique is one which causes absolutely no end or radial loading of the rotor relative to the stator.

Some dividing head manufacturers employ a mounting technique which rigidly clamps the rotor and the stator of the component under test to the dividing head. It is extremely difficult for an average skilled operator to mount a component in this type of dividing head without causing any radial and/or end loading due to misalignment. An extremely important precaution which must be taken with this mounting technique is to make sure that all mounting surfaces are entirely free from dirt. In fact, it is almost a requirement that this type of dividing head be operated in a dust-free room. In addition, unless the component under test is geometrically perfect, i.e., there is no eccentricity in the rotor, the rotor axis is perpendicular to the mounting surface of the stator, etc., several seconds or possibly minutes-of-arc error can be created.

The *Gertsch* mounting technique consists of rigidly clamping the rotor, and effectively floating the stator. The only precaution that the operator must take is to be careful not to apply physical force to the component through the wires attached to the stator.

Phase Angle Voltmeters

The Phase Angle Voltmeter (PAV) is ideal for synchro and resolver angular accuracy tests, since an external phase shifter is not required in order to compensate for the time phase shift angle of the component under test. In addition, large amounts of quadrature (fundamental null) can be tolerated without affecting the accuracy of the inphase null.

The *Gertsch* PAV has several features designed to aid synchro and resolver angular accuracy testing. For example, when using the "Proportional Voltage Gradient Method" 3 minutes-of-arc may correspond to, say, 19.6 mv. However, with the variable attenuator on the front panel of the PAV, 19.6 mv can be set up to equal full scale deflection on the 10 mv scale. Thus, the angular error at each 5° point can now be read directly on the 3-0-3 scale of the PAV. Other features include plug-in, ±5% band-pass, filters from 30 Hz to 10KHz, DC recorder output (± 1 V equals full scale on meter), variable reference angle from 0° to 360° continuously, variable attenuator — any voltage from 300 μV to 300V can be adjusted for full scale deflection, high quadrature rejection — overload lamp lights when the total input signal exceed 10 times the range selector position.

Synchro and Resolver Specifications

Synchro and resolver specifications have stabilized in the past decade. Before that, resolver specifications were, for all practical purposes, non-existent. The worst result of this was the confusion generated for several years by not having a set of *standard* equations relating input and output voltages to each other, as a function of shaft position.

The Polaris resolver specification Mil-R-21530 (D) was the first widely accepted publication covering both data-transmission and computational type resolvers. Shortly thereafter, a *draft* copy of Mil-S-22347 (data-transmission resolvers only) was issued which agreed with Mil-R-21530's equations; however, a few months later Mil-S-22666 (computational resolvers only) came out, and disagreed with Mil-R-21530's equations. As a result there was general agreement on the equations for data transmission type resolvers, and some disagreement on comuputational types. Fortunately, three years later Mil-R-23417 was released, which standardized on one set of equations for both types of resolvers.

The specification Mil-S-20708 (C) (data-transmission synchros only) has now been in use for over a decade, but still appears to be adeaquate.

Synchro and resolver standards, and inductive-type bridges are widely used for both military and commercial applications. In fact, a large number of these instruments are presently used in the most advanced guidance and fire control systems. However, neither the military nor the commercial specifications have been brought up-to-date in this respect, even though these units have been in wide use for over a decade. The currently available specifications show the obsolete "master" transmitter techniques, and the resistive-type bridges. The requirement for revising the current specifications is obvious, since at present there are no specifications which show what the values and tolerances of the various electrical parameters associated with these "new" units should be, and which circuit configurations are acceptable.

Future Trends

For obvious reasons, size 8 (approximately 0.8" in diameter) is fast becoming the most popular size component for new designs. Soon after that the trend will probably be toward size 5 components. With this trend toward smaller size components, will come a requirement for dividing heads which cause less and less radial and end loading.

Components with higher accuracies will of course become available; resolvers with one second-of-arc accuracy are not too difficult to imagine. Higher impedance values, and higher frequencies will also become more common.

The Hall-effect synchros and resolvers may go into production in a few years, assuming that the temperature sensitivity and output voltage limitation problems are solved. With them will come smaller sizes, lower voltages, and wider frequency ranges (D.C. to megacycles).

Major improvements in test equipment design are most likely to occur in the high frequency and high resolution area; however, major improvements in other areas such as dividing heads are also quite probable. It should be pointed out though that the test equipment available today is still several steps ahead of the majority of components currently available.

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APPLICATION NOTE NO. 2
SYNCHRO STANDARDS
THEORY AND APPLICATIONS

GENERAL DESCRIPTION

Gertsch Synchro Standards are precision transformer instruments, designed to simulate the "ideal" 3-wire electrical outputs (± 2 seconds-of-arc) of a Synchro Control (or Torque) Transmitter (CX). When driven by an external signal source the Synchro Standard generates, in precise angular increments (normally 5°), three voltages "120° in amplitude" from each other. One of the three output voltages is directly proportional to the sine of the angle displayed on the dial. Quadrant switching is provided to simulate operation over a full 360°.

Because of their very low output impedance and extremely low output impedance unbalance, Synchro Standards may be operated under normal load with no degradation of accuracy.

THEORY OF OPERATION

The following figure illustrates the basic circuitry common to all Gertsch Synchro Standards:

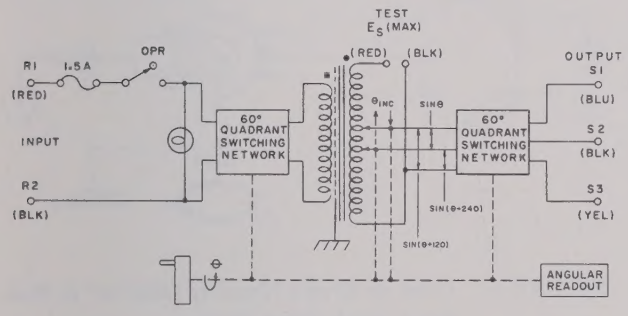

FIGURE 1. *Synchro Standard Block Diagram.*

The color coding of the binding posts, and the phase relationship of the output voltages relative to the input, conform to Mil-S-20708A. That is:

$$E_{S1\text{-}S3} = NE_{R2\text{-}R1} \sin \theta \quad (1)$$

$$E_{S3\text{-}S2} = NE_{R2\text{-}R1} \sin (\theta + 120°) \quad (2)$$

$$E_{S2\text{-}S1} = NE_{R2\text{-}R1} \sin (\theta + 240°), \quad (3)$$

where N is the transformation ratio of the Synchro Standard, and θ is positive for CCW rotation from electrical zero (EZ), when viewed from the shaft end. The standard outputs of a CX (and SS therefore) are shown in figure 2.

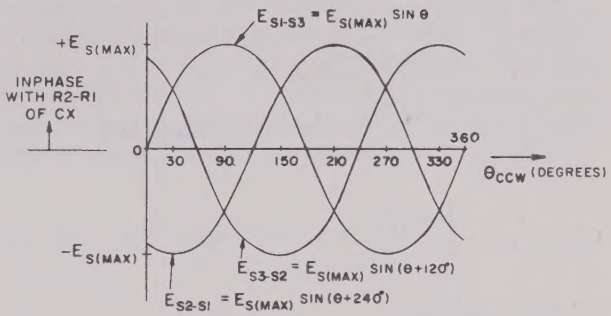

FIGURE 2. *Standard Synchro Control Transmitter (CX) Outputs as a Function of CCW Rotation From Electrical Zero (EZ).*

To convert the terminal identifications to the ARINC (Aeronautical Radio, Inc.) system nomenclature, the following relationships apply for 0 degree index reference, and positive rotation reference:

$$H = R1,\ C = R2,\ X = S1,\ Y = S3,\ Z = S2$$

The equivalent angular position of a Synchro Standard is uniquely and completely defined as a function of the ratio of the output voltages to each other: *

$$E_{S3\text{-}S1} = -NE_{R2\text{-}R1} \sin \theta \quad (4)$$

$$E_{S2\text{-}S1} = NE_{R2\text{-}R1} \sin (\theta + 240°). \quad (5)$$

Therefore, ideally:

$$A = \frac{\sin \theta}{\sin (\theta + 60°)}, \quad (6)$$

or

$$A = \frac{1}{2} - \frac{\sqrt{3}}{2} \cot (\theta + 60°), \quad (7)$$

where $A \equiv \frac{E_{S2\text{-}S1}}{E_{S3\text{-}S1}}$.

The relationship between a deviation in ratio and an equivalent angular error, is derived by differentiating equation (7) with respect to θ and converting to PPM and seconds-of-arc:

$$\Delta \theta_{(seconds)} = \frac{\Delta A_{(PPM)}}{4.20 \csc^2 (\theta + 60°)}. \quad (8)$$

The maximum effect of loading on the accuracy of a Synchro Standard may be calculated from the following approximation:

$$\Delta \theta_{MAX(sec)} \cong \pm \left[\text{Real Part Of} \left| \frac{Z_{O\ (unbalance)}}{Z_{SO}} \right| \times 10^5 - 2 \right], \quad (9)$$

where Z_{so} is the stator impedance of the load, with two stator terminals shorted together and with the rotor open, at 0.866 $E_{s(max)}$. Note, that $Z_{so} = \frac{1}{2} Z_{delta}$ when using delta loads.

The $Z_{o\ (unbalance)}$ for a Synchro Standard may be approximated at frequencies other than the nominal, by assuming the j component to be directly proportional to frequency.

The maximum line current from the primary and secondary of a Synchro Standard may be calculated from the following approximations:

$$I_{P\ (max)} = \frac{0.866\ NE_{S\ (max)}}{Z_{so}} \quad \text{at any angle, and} \qquad (10$$

$$I_{S\ (max)} = \frac{0.866 E_{S\ (max)}}{Z_{so}} \quad \text{at } 0°,\ 60°,\ 120°,\ 180°, \qquad (11)$$

240°, and 300°.

* For this reason, the deviation in transformation ratio from the nominal as a function of angle, does not affect the angular accuracy.

APPLICATIONS

Synchro Standards are generally used to transmit precise angular data to Synchro Control Differential Transmitters (CDX's) and Synchro Control Transformers (CT's). Most commercially available synchros of this type may be excited from a Gertsch Synchro Standard without affecting the 2-second accuracy.

Torque Transmitters (TX's) may also be simulated with Synchro Standards, in which case SS's could be used to transmit precise angular data to Torque Differential Transmitters (TDX's), Torque Differential Receivers (TDR's), and Torque Receivers (TR's).

The basic range of applications for SS's are best exemplified by the following figures:

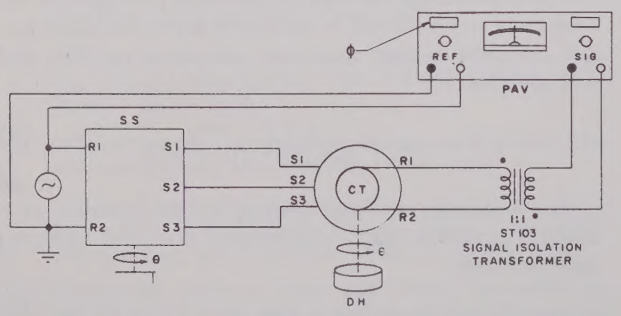

FIGURE 3. *CT Angular Accuracy Test.**

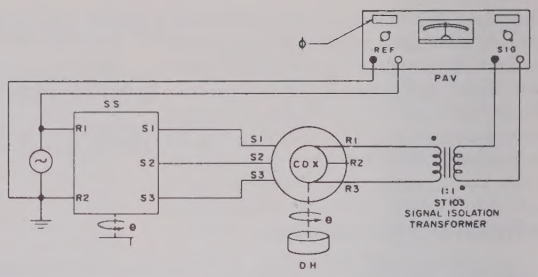

FIGURE 4. *CDX (stator) Angular Accuracy Test.**

* Note that either the "Proportional Voltage Nulling" or the Proportional Voltage Gradient" method may be used. See Application Note # 9.

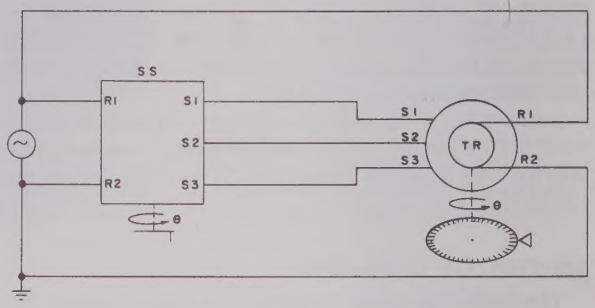

FIGURE 5. *Using an SS as a "Dummy Director" to Position a Torque Receiver (Repeater).*

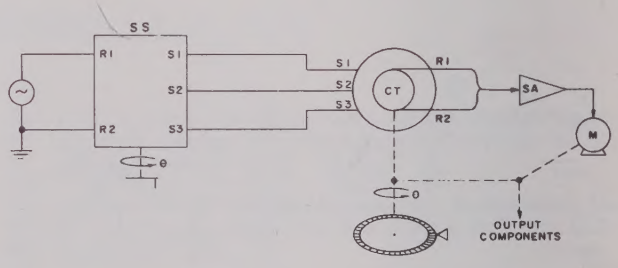

FIGURE 6. *Using an SS as a "Dummy Director" to Position a Single-Speed Servo.*

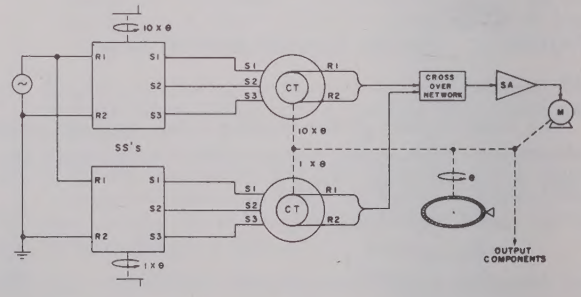

FIGURE 7. *Using Two SS's as a "Dummy Director" to Position a Two-Speed Servo.***

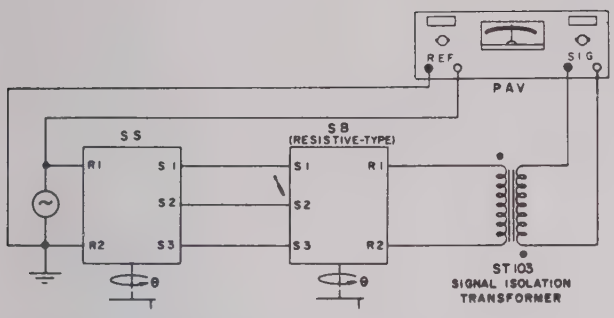

FIGURE 8. *Calibrating Lower-Precision Resistive-Type Synchro Bridges with an SS.* Note that the error voltage from the SB may be converted to an equivalent angular error (See Application Note # 6).

** Note that the accuracy of the SS is 0.2 seconds, due to the 10:1 gear ratio.

ADVANTAGES

The unique passive circuit employed in the Gertsch Synchro Standard replaces the complex maze of instruments and inter-connections typically used for performing angular accuracy tests on Synchro Control Transformers and Control Differential Transmitters (stator). The conventional method requires the use of a "Master" Synchro Control Transmitter, a Phase Angle Voltmeter, a Dividing Head, a Bridging Transformer, and a Synchro Bridge. A Gertsch Synchro Standard can replace *all* of these instruments and perform the tests with increased accuracy, speed, less initial cost, less space, and with less chance of operator error.

APPLICATION NOTE NO. 3

SYNCHRO BRIDGES
THEORY AND APPLICATIONS

GENERAL DESCRIPTION

A Gertsch Synchro Bridge is a precision transformer instrument, which produces an error voltage when its angular position differs from the equivalent angular position of the three-wire signal applied to its input. This equivalent angular position is uniquely and completely defined as a function of the ratio of the input signal voltages to each other. In some respects, a synchro bridge may be considered to be analogous to an "ideal" incremental (normally 5° ± 2-seconds) control transformer (CT); however, unlike a CT, a synchro bridge does not have the same output error voltage gradient (sensitivity—volts/degree) at each position, unless a voltage gradient compensating network is added.

THEORY OF OPERATION

The basic circuitry common to all Gertsch Synchro Bridges is illustrated in the following figure (note that *uncompensated* units do *not* have a scaling transformer):

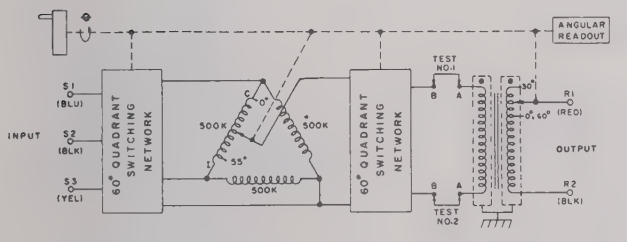

FIGURE 9. *Compensated Synchro Bridge Block Diagram.*

The color coding of the binding posts, and the phase relationship of the output relative to the input voltages, conform to Mil-S-20708B.

The conversion of the terminal identifications to the ARINC system is the same as for the SS's (See Application Note # 2).

Since the net circulating current around the delta is zero at null, it may be seen with the aid of figure 9 that, ideally, for the position of the tapped arm shown, and for any angle (Θ) between 0° and 60°:

$$A = \frac{-E_{s(max)} \sin \Theta}{E_{s(max)} \sin (\Theta + 240°)} = \frac{\sin \Theta}{\sin (\Theta + 60°)}, \quad (12)$$

where $A \equiv \frac{OC}{IC}$.

With the use of trigonometric identities, it can be shown that equation (12) is equal to:

$$A = \frac{1}{2} - \frac{\sqrt{3}}{2} \cot (\Theta + 60°), \quad (13)$$

which is the equation usually found in synchro specifications.

To find the equivalent angular error of a bridge, caused by a tap ratio deviation at a particular angle, equation (13) is differentiated with respect to Θ and converted to PPM and seconds-of-arc:

$$\Delta\Theta_{(seconds)} = \frac{\Delta A_{(ppm)}}{4.20 \csc^2 (\Theta + 60°)}. \quad (14)$$

The equivalent angular error caused by an input impedance unbalance may be calculated from the following approximation:

$$\Delta\Theta_{MAX\ (sec)} \cong \text{Real Part Of } \left(\frac{4 Z_{SS}(\Omega) \cdot Z'_{UNBALANCE}(\%)}{3 Z'\ (K\Omega)} \right), \quad (15)$$

where Z′ is the average value of $Z_{ARM\ (max)}$ and $Z_{ARM\ (min)}$, and $Z'_{UNBALANCE}$ is equal to the difference between $Z_{ARM\ (max)}$ and $Z_{ARM\ (min)}$ divided by Z′. Z_{SS} is equal to the stator (S1-S3 to S2) output impedance of the synchro under test, with the rotor shorted.

A Figure-of-Merit may be defined for a synchro bridge, relating input impedance and input impedance unbalance:

$$\gamma \equiv \frac{Z' \ (K\Omega)}{Z'_{\text{unbal}} \ (\%)/\sin(2\theta + \alpha)}. \qquad (16)$$

Thus,

$$\Delta\theta_{MAX(sec)} \cong \text{Real Part Of } \left(\frac{4\,Z_{SS}\,(\Omega)}{3\gamma}\right). \qquad (17)$$

Note, that the sin $(2\theta + \alpha)$ factor allows $\Delta\theta$ to deviate from a pure sinusoidal error curve. α is an arbitrary angular constant.

From figures 2 and 9 it may be seen that:

$$V_{R2-R1} = -\left[\sin\theta + A\sin(\theta + 240°)\right] E_{s(max)}, \qquad (18)$$

where $E_{s(max)}$ is the maximum RMS stator output voltage of the synchro under test, and V_{R2-R1} is out-of-phase with E_{R2-R1} of the synchro under test for a positive angular error (θ_{elect} greater than θ_{shaft}).

To find the equivalent angular error of the output error voltage from a bridge at a particular angle, equation (18) is differentiated with respect to θ:

$$\frac{dV_{R2-R1}}{d\theta} = -\left[\cos\theta + A\cos(\theta + 240°)\right] E_{s(max)}. \qquad (19)$$

Substituting equation (12) for A, disregarding the negative sign, and for fairly large changes ($\Delta\theta = 5°$, gives a ΔV error of approximately $+0.13\%$ at any angle θ):

$$\frac{\Delta V_{R2-R1}}{\Delta\theta} = \left[\cos\theta - \sin\theta\cot(\theta + 60°)\right] E_{s(max)}, \qquad (20)$$

where $\Delta\theta$ is in radians, and θ is the angle of the bridge.

Converting $\Delta\theta$ from radians to seconds, and letting

$$K \equiv \left[\cos\theta - \sin\theta\cot(\theta + 60°)\right] \qquad (21)$$

yields:

$$\frac{\Delta V_{R2-R1\ (volts)}}{\Delta\theta_{(seconds)}} = 4.85 \times 10^{-6}\, K\, E_{s(max)}. \qquad (22)$$

Examination of equation (21) reveals that K is maximum (1.000) at 0° and 60°, and minimum (0.866) at 30°. Therefore, the error voltage gradient $\frac{\Delta V}{\Delta\theta}$ is *not* the same at each angular position of the bridge, for a conventional delta bridge.

A voltage gradient compensating network (scaling transformer) may be added to the output of a synchro bridge, which will adjust K to the same value at each angle and, therefore, make $\frac{\Delta V_{R2-R1}}{\Delta\theta}$ independent of θ. This type of delta bridge (with a voltage gradient compensating network) is used with what is referred to as the "Proportional Voltage Gradient Method" of testing by Mil-S-20708B.

Basically, K may be adjusted to the same value at each angle with the use of a scaling transformer that has a special tap for each 5° position of the bridge.

Usually, it is desirable to adjust K to 1.000 (K′) at each position, in order to simplify equation (22). This is accomplished by locating the taps so that the transformation ratio of the scaling transformer at each angle is:

$$\text{T. R.} = \frac{K'}{K} = \frac{1.000}{K}. \qquad (23)$$

A compensated synchro bridge with a K′ equal to 1.000 is equivalent to a Synchro Control Transformer (CT) with a transformation ratio equal to 1.155*.

1.000 may now be substituted for K in equation (22); therefore, $\frac{\Delta V_{R2-R1}}{\Delta\theta}$ becomes:

$$\frac{\Delta V_{R2-R1(volts)}}{\Delta\theta_{(seconds)}} = 4.85 \times 10^{-6} E_{s(max)}. \qquad (24)$$

* The transformation ratio of a CT, as defined by Mil-S-20708B, is equivalent to the rotor voltage at maximum coupling divided by 0.866 $E_{s(max)}$.

APPLICATIONS

Gertsch Synchro Bridges are ideally suited for checking both Torque and Control Transmitters and Differentials (rotor) as components, or simulating an incremental Control Transformer in a servo loop.

Compensated Synchro Bridges are required when using the "Proportional Voltage Gradient Method," and are highly recommended whenever an isolated output is required. A compensated unit should also be employed when using a bridge to drive a servo, because of the isolated output and constant error voltage gradient. For best results, the output of an unisolated bridge should never be grounded.

The following figures illustrate a few basic applications.

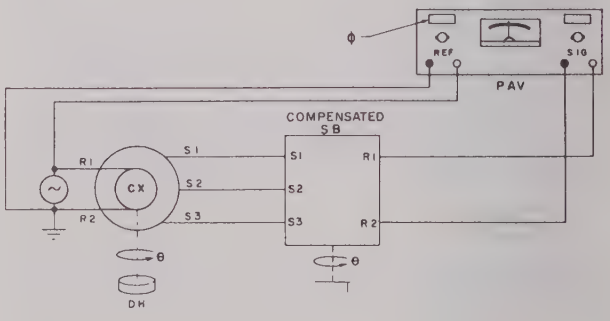

FIGURE 10. *CX (or TX) Angular Accuracy Test.* Note, that the "Proportional Voltage Gradient Method" may be used, since the bridge is compensated. See Application Note # 9.

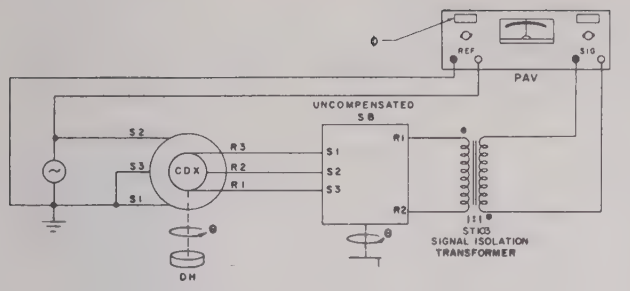

FIGURE 11. *CDX (or TDX) (Rotor) Angular Accuracy Test.* Note, that the "Proportional Voltage Nulling Method" *must* be used, since the bridge is not compensated. See Application Note # 9.

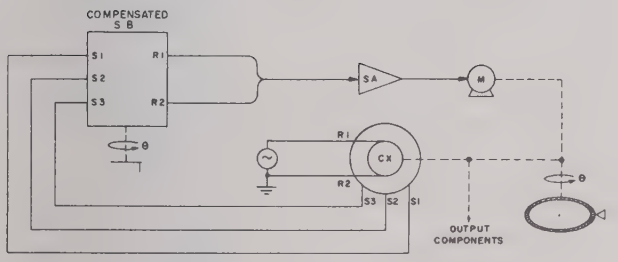

FIGURE 12. *Using an SB to Position a Single-Speed Servo.*

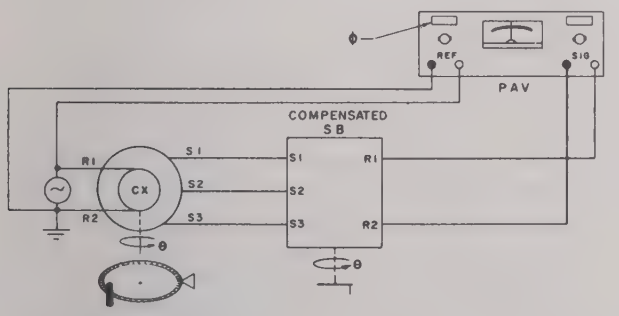

FIGURE 13. *Calibrating Lower-Precision "Dummy Directors" with an SB.* Note, that angles other than multiples of 5° may be tested by utilizing the PAV to read angles in terms of "error" voltage. Accuracy is ± 3% of angle represented by full-scale on the PAV. See Application Note # 9.

ADVANTAGES

Gertsch Synchro Bridges have several advantages over conventional 10K resistive-type bridges. Many of these advantages are due to their inherently high input impedance, low output impedance and low phase shift, plus the high tap ratio accuracy (equivalent to ± 2-seconds). The high input impedance (500K) of these bridges allows the user to measure the equivalent angular position of a synchro under *actual* load without causing a significant angular error due to bridge loading. Since the output impedance is approximately three orders-of-magnitude less than 10K resistive-type bridges, the bridge is essentially insensitive to pickup and loading errors. This very low output impedance, and the extremely low phase shift of these bridges makes them ideal for driving low impedance loads. In addition, compensated units have a built-in, double-shielded scaling transformer with an isolated output.

APPLICATION NOTE NO. 4

RESOLVER STANDARDS THEORY AND APPLICATIONS

GENERAL DESCRIPTION

Gertsch Resolver Standards are precision transformer instruments, designed to simulate the "ideal" electrical outputs (± 2 seconds-of-arc) of a Resolver Control Transmitter (RX). When driven by an external signal source the Resolver Standard generates, in precise angular increments (normally 5°), two isolated voltages. One of the output voltages is directly proportional to the sine of the angle displayed on the dial, and the other is proportional to the cosine. Both outputs are isolated from each other. Quadrant switching is provided to simulate operation over a full 360°.

Because of their very low output impedance and extremely low output impedance unbalance, Resolver Standards may be operated under normal load with no degradation of accuracy.

THEORY OF OPERATION

The following figure illustrates the basic circuitry common to all Gertsch Resolver Standards:

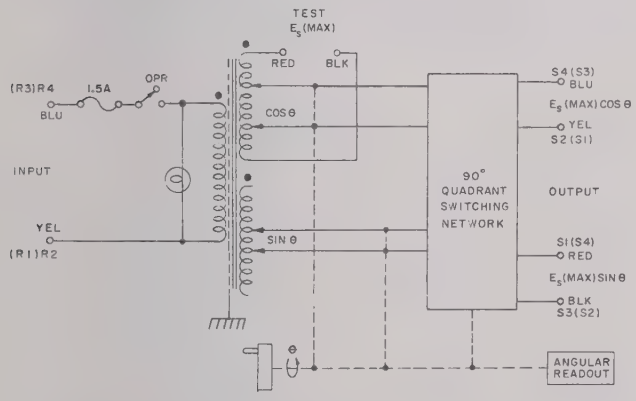

FIGURE 14. *Resolver Standard Block Diagram.*

Operational versatility of the Resolver Standard is increased by the use of two sets of terminal identifications. The input identification selected [R2-R4 or (R3)-(R1)] determines the output identification markings to be used by the operator. The binding posts are color coded per Mil-S-22347 and Mil-R-21530D, for the terminal identifications not parenthetically enclosed, i.e. R2-R4 input.

The phase relationships of the output voltages relative to the input voltage, conform to Mil-S-22347 and Mil-R-21530D for both sets of terminal identifications. That is:

$$E_{S1\text{-}S3} = N\,(E_{R1\text{-}R3}\ \cos\theta - E_{R2\text{-}R4}\ \sin\theta) \qquad (25)$$

$$E_{S2\text{-}S4} = N\,(E_{R2\text{-}R4}\ \cos\theta + E_{R1\text{-}R3}\ \sin\theta), \qquad (26)$$

where N is the transformation ratio of the Resolver Standard, and θ is positive for CCW rotation from electrical zero (EZ), when viewed from the shaft end. The standard outputs of an RX (and RS therefore) are shown in figure 15 for R2-R4 excitation.

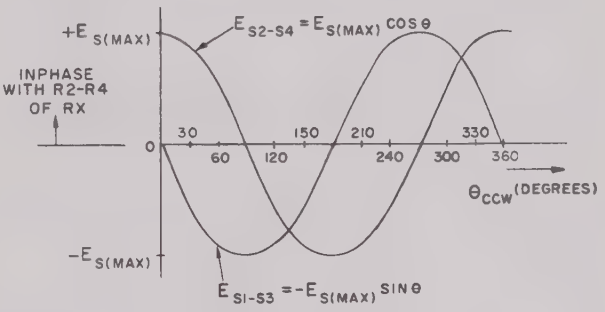

FIGURE 15. *Standard Resolver Control Transmitter (RX) Outputs as a Function of CCW Rotation From Electrical Zero (EZ), With R2-R4 Excited.*

The equivalent angular position of a Resolver Standard is uniquely and completely defined as a function of the ratio of the output voltages to each other:*

$$E_{S3\text{-}S1} = NE_{R2\text{-}R4} \sin\theta \qquad (27)$$

$$E_{S2\text{-}S4} = NE_{R2\text{-}R4} \cos\theta. \qquad (28)$$

Therefore, ideally:

$$A = \tan\theta, \qquad (29)$$

where $A \equiv \dfrac{E_{S3\text{-}S1}}{E_{S2\text{-}S4}}$.

The relationship between a deviation in ratio and an equivalent angular error is derived by differentiating equation (29) with respect to θ and converting to PPM and seconds-of-arc.

$$\Delta\theta_{\text{(seconds)}} = \frac{\Delta A_{\text{(PPM)}}}{4.85\ \sec^2\theta}. \qquad (30)$$

The maximum effect of loading on the accuracy of a Resolver Standard may be calculated from the following approximation:

$$\Delta\theta_{MAX\,(sec)} \cong \pm \left[\text{Real Part Of} \left| \frac{Z_{o\ (unbalance)}}{Z_{so}} \right| \times 10^5 - 2 \right], \qquad (31)$$

where Z_{so} is the stator impedance of the load, with the rotor open, at 0.707 $E_{s\,(max)}$.

The $Z_{o\,(unbalance)}$ of a Resolver Standard may be approximated at frequencies other than the nominal, by assuming the j component to be directly proportional to frequency.

The maximum line current from the primary and secondary of a Resolver Standard may be calculated from the following approximations:

$$I_{P\,(max)} \cong \frac{NE_{S\,(max)}}{Z_{so}} \text{ at any angle, and} \qquad (32)$$

$$I_{S\,(max)} \cong \frac{E_{S\,(max)}}{Z_{so}} \text{ at } 0°,\ 90°,\ 180°, \text{ and } 270°. \qquad (33)$$

* For this reason, the deviation in transformation ratio from the nominal as a function of angle does not affect the angular accuracy.

APPLICATIONS

Resolver Standards are generally used to transmit precise angular data to Resolver Control Differential Transmitters (RD's) and Resolver Control Transformers RC's). Most commercially available resolvers of this type may be excited from a Gertsch Resolver Standard without affecting the 2-second accuracy.

The basic range of applications for RS's are best exemplified by the following figures:

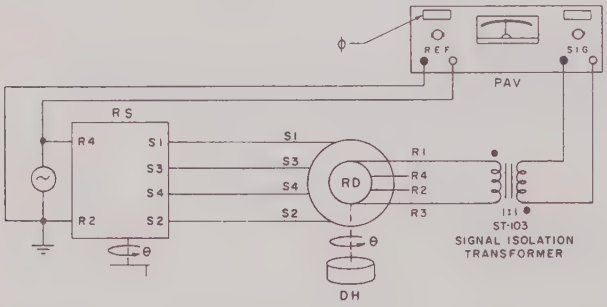

FIGURE 16. *RD (stator) (or RC) Angular Accuracy Test.* Note, that either the "Proportional Voltage Nulling" or the "Proportional Voltage Gradient" method may be used. See Application Note # 9.

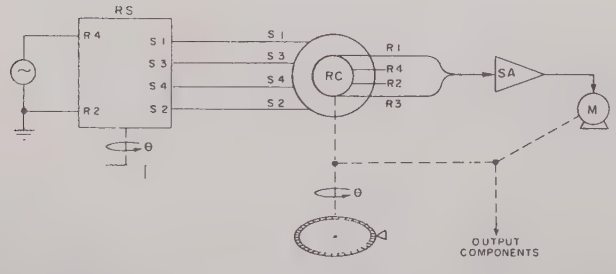

FIGURE 17. *Using an RS as a "Dummy Director" to Position a Single-Speed Servo.*

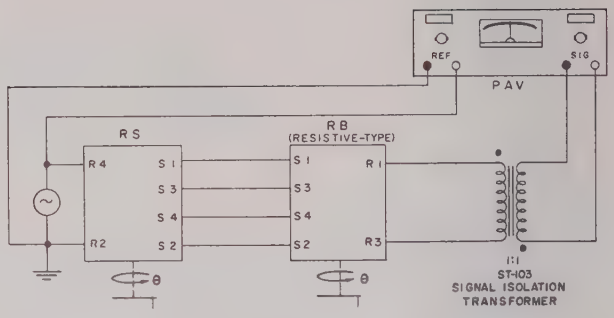

FIGURE 18. *Calibrating Lower-Precision Resistive-Type Resolver Bridges.* Note, that the error voltage from the RB may be converted to an equivalent angular error (See Application Note # 6).

ADVANTAGES

The unique passive circuit employed in the Gertsch Resolver Standard replaces the complex maze of instruments and inter-connections typically used for performing angular accuracy tests on Resolver Control Transformers and Control Differential Transmitters (stator). The conventional method requires the use of a "Master" Resolver Control Transmitter, a Phase Angle Voltmeter, a Dividing Head, a Bridging Transformer, and a Resolver Bridge. A Gertsch Resolver Standard can replace *all* of these instruments and perform the tests with increased accuracy, speed, less initial cost, less space, and with less chance of operator error.

APPLICATION NOTE NO. 5

RESOLVER STANDARDS THEORY AND APPLICATIONS

GENERAL DESCRIPTION

A Gertsch Resolver Bridge is a precision transformer instrument, which produces an error voltage when its angular position differs from the equivalent angular position of the signal applied to its input. This equivalent angular position is uniquely and completely defined as a function of the ratio of the two input signal voltages to each other. In some respects, a resolver bridge may be considered to be analogous to an "ideal" incremental (normally 5° ±2-seconds) control transformer (RC). All Gertsch Resolver Bridges have the same output error voltage gradient (sensitivity — volts/degree) at each position, since the unique design employed is inherently compensated.

THEORY OF OPERATION

The basic circuitry common to all Gertsch Resolver Bridges is illustrated in the following figure:

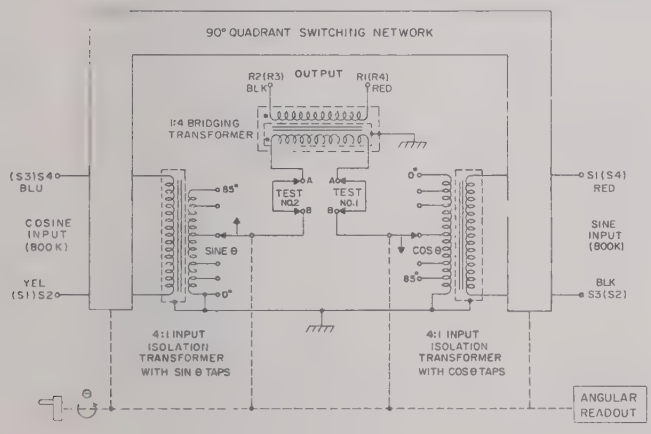

FIGURE 19. *Compensated Resolver Bridge Block Diagram.*

Operational versatility of the Resolver Bridge is increased by the use of two sets of terminal identifications. The output identification selected [R3-R1 or (R4)-(R2)] determines the input identification markings to be used by the operator. The binding posts are color coded per Mil-S-22347 and Mil-R-21530D, for the terminal identifications not parenthetically enclosed, i.e. R3-R1 output.

The phase relationship of the output voltage relative to the input voltages, conform to Mil-S-22347 and Mil-R-21530D for both sets of terminal identifications.

At null, when θ is between 0° and 90°:

$$E_{S2\text{-}S4} \cdot A_{R3} = E_{S3\text{-}S1} \cdot A_{R1}, \tag{34}$$

where $A \equiv \frac{OC}{IC}$, and A_{R3} and A_{R1} are the ratios of the two arms associated with the output terminals R3 and R1 respectively.

Since $E_{S2\text{-}S4}$ and $E_{S3\text{-}S1}$ are cosine and sine functions respectively, it is clear that the ratio of A_{R3} to A_{R1} must equal the tangent of θ.

A straightforward and advantageous solution is to let A_{R3} and A_{R1} equal the sine and cosine respectively:

$$A_{R3} = \sin\theta \tag{35}$$

$$A_{R1} = \cos\theta. \tag{36}$$

Therefore,

$$\frac{A_{R3}}{A_{R1}} = \tan\theta. \tag{37}$$

To find the equivalent angular error caused by a tap ratio deviation by both A_{R3} and A_{R1}, equation (37) must first be solved for θ:

$$\theta = \tan^{-1}\frac{A_{R3}}{A_{R1}}. \tag{38}$$

Now, since both A_{R3} and A_{R1} are variables, equation (38) must be differentiated by parts, which yields:

$$\left|\frac{\partial\theta_{R3}}{\partial A_{R3}}\right|_{A_{R1}=\text{constant}} = \cos\theta, \tag{39}$$

and

$$\left|\frac{\partial\theta_{R1}}{\partial A_{R1}}\right|_{A_{R3}=\text{constant}} = -\sin\theta. \tag{40}$$

However, for small changes:

$$\Delta\theta_{TOTAL} = \Delta\theta_{R3} + \Delta\theta_{R1} \tag{41}$$

Therefore, combining equations (39), (40), and (41), and converting $\Delta\theta$ from radians to seconds and ΔA to parts-per-million yields:

$$\Delta\theta_{TOTAL\ (seconds)} = 0.206\ (\Delta A_{R3\ (ppm)}\cos\theta - \Delta A_{R1\ (ppm)}\sin\theta), \tag{42}$$

where ΔA is positive if the measured value of A is larger than the theoretical value.

The purpose of the 4:1 step down ratio for the input isolation transformers is to reduce the reflected impedance from secondary to primary by a factor of 16. Consequently, it is possible to obtain higher values of input impedance.

Differences in transformation ratio between the two input isolation transformers will result in a $\pm$ ($\triangle \Theta_{MAX}$) sin 2Θ error function; therefore, they must be matched to within a few parts-per-million.

The equivalent angular error caused by an input impedance unbalance may be calculated from the following approximation:

$$\triangle\Theta_{MAX\ (sec)} \cong \text{Real Part Of} \left(\frac{Z_{SS}(\Omega) \cdot Z'_{UNBALANCE}\ (\%)}{Z'\ (K\Omega)}\right), \qquad (43)$$

where Z' is the average value of Z_{S3-S1} and Z_{S2-S4}, and $Z'_{UNBALANCE}$ is equal to the difference between Z_{S3-S1} and Z_{S2-S4} divided by Z'. Z_{SS} is equal to the stator (S1-S3, or S2-S4) output impedance of the resolver under test, with the rotor shorted.

A Figure-of-Merit may be defined for a resolver bridge, relating input impedance and input impedance unbalance:

$$\gamma \equiv \frac{Z'\ (K\Omega)}{Z'_{\ unbal}\ (\%)\ /\sin 2\Theta}. \qquad (44)$$

Thus,

$$\triangle\Theta_{MAX\ (sec)} \cong \text{Real Part Of} \left(\frac{Z_{SS}\ (\Omega)}{\gamma}\right). \qquad (45)$$

Note, that the sin 2Θ factor allows $\triangle\Theta$ to deviate from a pure sinusoidal error curve.

From figures 15 and 19 it may be seen that:

$$V_{R3-R1} = [(1-A)\ \sin\Theta - A\cos\Theta]E_{s(max)}, \qquad (46)$$

where $E_{s(max)}$ is the maximum RMS stator output voltage of the resolver under test, and V_{R3-R1} is inphase with E_{R2-R4} of the resolver under test for a positive angular error (Θ_{elect} greater than Θ_{shaft}).

To find the equivalent angular error of the output error voltage from a bridge at a particular angle, equation (46) is differentiated with respect to Θ:

$$\frac{dV_{R3-R1}}{d\Theta} = (A_{R1}\cos\Theta + A_{R3}\sin\Theta)E_{s(max)}. \qquad (47)$$

Substituting cos Θ and sin Θ for A_{R1} and A_{R3} respectively, and for fairly large changes ($\triangle\Theta = 5°$ gives a $\triangle V$ error of approximately $+0.13\%$ at any angle Θ):

$$\frac{\triangle V_{R3-R1}}{\triangle\Theta} = E_{s(max)}, \qquad (48)$$

where $\triangle\Theta$ is in radians. Note, that $\frac{\triangle V}{\triangle\Theta}$ is independent of the angular setting of the bridge; therefore, a voltage gradient compensating network is not required. This is the advantage of letting A_{R3} and A_{R1} equal the sine and cosine respectively. Note also, that the transformation ratio is unity.

Converting $\triangle\Theta$ from radians to seconds yields:

$$\frac{\triangle V_{R3-R1\ (volts)}}{\triangle\Theta\ (seconds)} = 4.85\times10^{-6}\ E_{s(max)}. \qquad (49)$$

APPLICATIONS

Gertsch Resolver Bridges are ideally suited for checking Control Transmitters and Differentials (rotor), or simulating an incremental Control Transformer in a servo loop.

As previously mentioned, all Gertsch Resolver Bridges are compensated and have an isolated output. Therefore, they are ideal for applications requiring a constant error voltage gradient, and/or a balanced output. Angular accuracy tests employing the "Proportional Voltage Gradient Method," or driving a servo, are two typical applications requiring a compensated bridge. An isolated output is required wherever a single-ended load is to be applied.

The following figures illustrate a few basic applications.

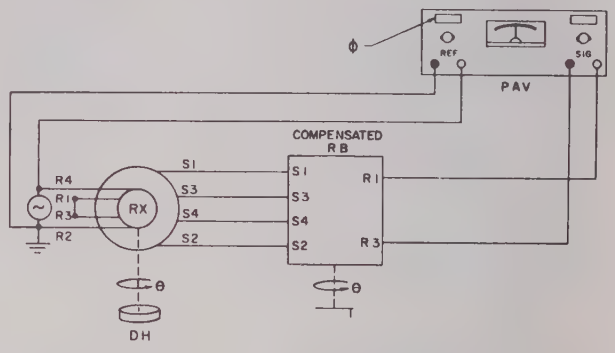

FIGURE 20. *RX Angular Accuracy Test.**

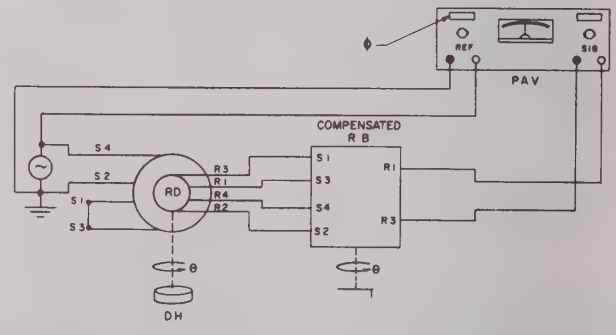

FIGURE 21. *RD (rotor) Angular Accuracy Test.**

* Note, that either the "Proportional Voltage Nulling" or the "Proportional Voltage Gradient" method may be used. See Application Note # 9.

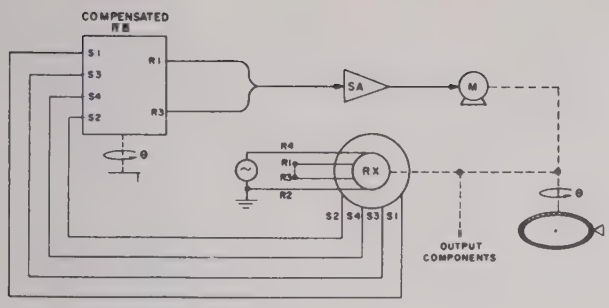

FIGURE 22. *Using an RB to Position a Single-Speed Servo.*

ADVANTAGES

Gertsch Resolver Bridges have several advantages over conventional 10K resistive-type bridges. Many of these advantages are due to their inherently high input impedance, low output impedance and low phase shift, plus the high tap ratio accuracy (equivalent to $\pm$ 2-seconds). The high input impedance (800K) of these bridges allows the user to measure the equivalent angular position of a resolver under *actual* load without causing a significant angular error due to bridge loading. Since the output impedance is approximately three orders-of-magnitude less than 10K resistive-type bridges, the bridge is essentially insensitive to pickup and loading errors. This very low output impedance and the extremely low phase shift of these bridges makes them ideal for driving low impedance loads. In addition, all units have a built-in, double-shielded bridging transformer with an isolated output

APPLICATION NOTE NO. 6

An Analysis of Synchro and Resolver Bridges

by

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First Edition 1-19-62

INTRODUCTION

With the increasing accuracies of synchros and resolvers used in data-transmission servo systems, it is becoming essential for the person involved with synchro and resolver bridges to fully understand the details associated with their operation.

Basically, a synchro or a resolver bridge is a passive network which produces an error voltage, when its angular position differs from the equivalent angular position of the signal applied to its input. This equivalent angular position is uniquely and completely defined as a function of the ratio of the input signal voltages to each other. In some respects, a bridge may be considered to be analogous to a control transformer; however, unlike control transformers, most bridges do not have the same output error voltage gradient at each position, unless a voltage gradient compensating network is added.

The analysis that follows is divided into two parts, namely, synchro bridges and resolver bridges. Each part discusses the relationship between the angular position of the bridge, and the ideal tap position of the bridge null detector output terminals. Next, the equivalent angular error caused by a ratio deviation from the ideal tap position of the output terminals is expressed as a function of the ideal angular position of the bridge. In addition, the effective angular error caused by a bridge impedance unbalance is discussed. Also, the equivalent angular error of the output error voltage from a bridge is expressed as a function of the angular position of the bridge, and the maximum output voltage of the unit under test. In conclusion, it is shown how voltage gradient compensating networks may be designed in order to make the initial slope of the error voltage gradient independent of the angular position of the bridge. The equivalent angular error of a unit under test may, therefore, be expressed only as a function of the bridge output error voltage and the maximum output voltage of the unit.

It will be noted that, in general, the bridges which are analyzed in the following sections are of the precision ratio-transformer type; however, the analysis is also valid for bridges which employ the resistive voltage divider technique.

The Gertsch ratio-transformer (RatioTran) types of bridges have several characteristics which offer distinct advantages over resistive type bridges. They are as follows:

1. High Ratio Accuracies: Standard synchro and resolver bridges in the Gertsch line have a ratio accuracy equivalent to 2 seconds-of-arc.

2. High Stability: The ratio accuracy of the RatioTran type of bridge is primarily based upon the use of a toroidal autotransformer which is not affected by age or environmental conditions.

3. High Input Impedance: At the designed operating frequency, (selected to obtain maximum input impedance consistant with minimum variations as a function of voltage) the input impedance of RatioTran synchro and resolver bridges is approximately 500K ohms and 800K ohms respectively.

Since the input impedance of RatioTran bridges is several times higher than the resistive type bridges, either the short-circuit output impedance (Z_{ss}) of the unit under test or the input impedance of the RatioTran bridge may be unbalanced by a proportionally larger amount, for a given Z_{ss} and a given equivalent angular error. This higher input impedance also allows the equivalent angular error caused by an unbalanced Z_{ss} to be determined separately under actual loading conditions, rather than 10K ohms of pure resistance, by merely paralleling the desired load with the input to the bridge.

In general, the required accuracy of resolver bridges is higher than synchro bridges, since resolvers currently being produced have accuracies as high as 10 seconds, while most synchros are presently limited to about 90 seconds. Therefore, the input impedance of synchro bridges may be lower, and/or the input impedance unbalance may be higher.

4. Low Output Impedance: The output impedance of RatioTran bridges may be considered to be negligible for most applications. In contrast to this, uncompensated 10K ohm resistive type synchro and resolver bridges have an output impedance which varies with the angular setting from 0 to 2.5K ohms, and from 0 to 5K ohms respectively. The output impedance of compensated resistive type synchro and resolver bridges varies from approximately 8.8K to 2.4K ohms, and from 18K to 4.6K ohms respectively.

5. Low Phase Shift: The phase error in the output of the RatioTran bridge is a function of frequency and angular position, but is nominally 0.05 milliradians which may normally be neglected.

Gertsch Synchro and Resolver Standards, which simulate ideal CX's and RX's respectively, also employ the ratio-transformer (RatioTran) technique to obtain a high degree of angular accuracy (2 seconds), and an effective output impedance unbalance as low as $.01 \angle 5.7°$ ohms.

PART I
SYNCHRO BRIDGES

RATIOTRAN SYNCHRO BRIDGE

A RatioTran Synchro Bridge is illustrated in figure 1. This delta type of bridge (without a voltage gradient compensating network) is used with what is referred to as the "Proportional Voltage Nulling Method" of testing by MIL-S-20708A[1].

Figure 2 illustrates the phase relationships of the output voltages of a standard synchro control transmitter (CX), which are normally applied to the three input terminals of a synchro bridge.

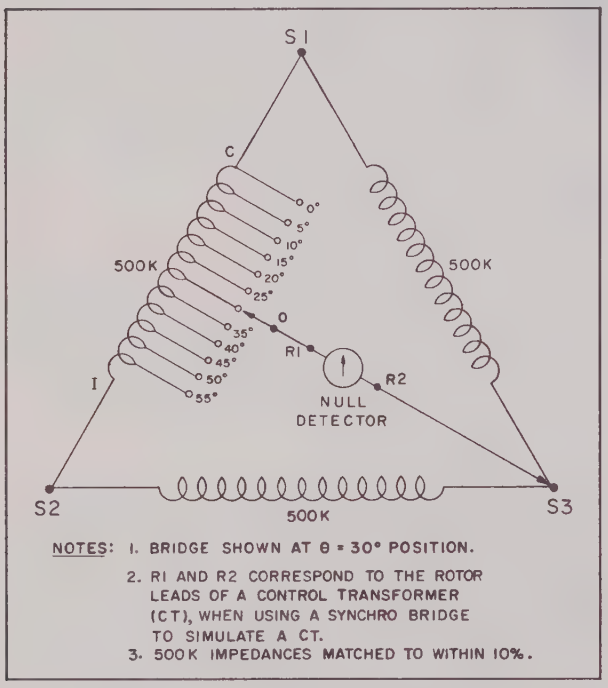

FIGURE 1. RatioTran Synchro Bridge.

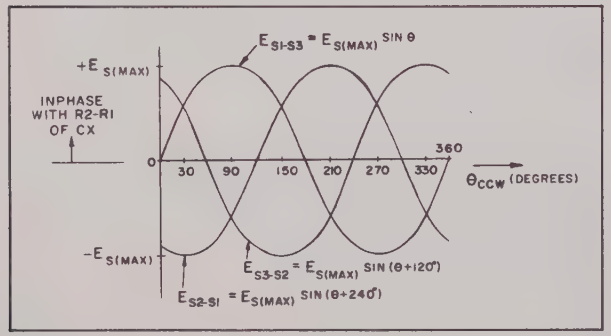

FIGURE 2. Standard Synchro Control Transmitter (CX) Outputs as a Function of CCW Rotation From Electrical Zero (EZ).

IDEAL TAP RATIOS

Since the net circulating current around the delta is zero at null, it may be seen with the aid of figure 2 that, ideally, for the position of the tapped arm shown, and for any angle (θ) between 0° and 60°:

$$A = \frac{-E_{s(max)} \sin \theta}{E_{s(max)} \sin (\theta + 240°)} = \frac{\sin \theta}{\sin (\theta + 60°)}, \quad (1)$$

where $A \equiv \frac{OC}{IC}$.

In the following analysis, θ is generally restricted to angles between 0° and 60°. However, values of θ not within this range may be converted by subtracting a multiple of 60°, since the bridge characteristics repeat every 60°.

With the use of trigonometric identities, it can be shown that equation (1) is equal to:

$$A = \frac{1}{2} - \frac{\sqrt{3}}{2} \cot (\theta + 60°), \quad (2)$$

which is the equation usually found in synchro specifications.

It is interesting to note that the position of the tapped arm, as listed in table I, must always be between the two terminals with the largest voltage.

TABLE I

Positions of Elements Within a Synchro Bridge From 0° to 360°

Synchro Bridge Angle θ (Degrees)	Position of Tapped Arm (C to I)	Position of Null Detector Terminals	
		R1	R2
0-60	S1 - S2	Tapped arm	S3
60-120	S3 - S1	S2	Tapped arm
120-180	S2 - S3	Tapped arm	S1
180-240	S1 - S2	S3	Tapped arm
240-300	S3 - S1	Tapped arm	S2
300-360	S2 - S3	S1	Tapped arm

In figure 3, equation (2) is plotted as a function of θ. The plotted curve shows that the terminal linearity of the tapped arm is approximately 1.8%. Therefore, if the taps were equally spaced, an equivalent maximum angular error of 1.1° ($0.018 \times 60° = 1.1°$) would result.

The precise values of A, for 5° increments, are listed in table II. Note, that at any angle:

$$A_{(\theta)} + A_{(60° - \theta)} = 1.0000000. \quad (3)$$

TABLE II

Precise Values of A for a Synchro Bridge in 5° Increments

Synchro Bridge Angle, θ (Degrees)	$A \equiv \frac{OC}{IC}$
0	.0000000
5	.0961657
10	.1847926
15	.2679492
20	.3472963
25	.4242326
30	.5000000
35	.5757674
40	.6527037
45	.7320508
50	.8152074
55	.9038343
60	1.0000000

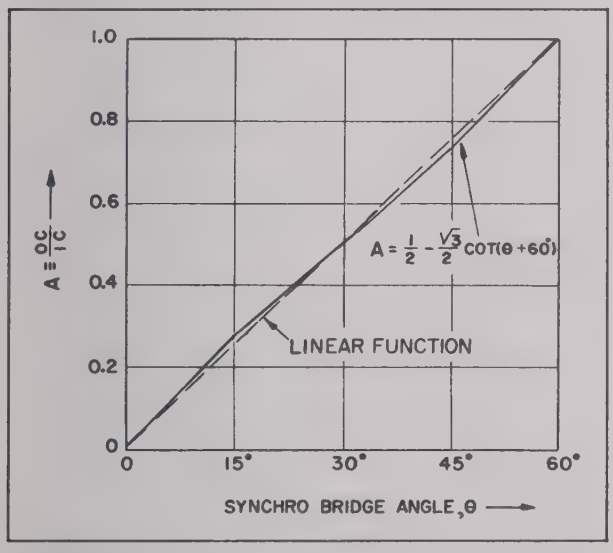

FIGURE 3. A as a Function of Synchro Bridge Angle, θ.

SINGLE-ARM SYNCHRO BRIDGE

Equation (2) may be solved for θ, which results in:

$$\theta = \cot^{-1}\left(\frac{1-2A}{\sqrt{3}}\right) - 60°. \qquad (4)$$

This equation is sometimes useful, when using a precision decade ratio-transformer as a high-resolution, and high accuracy synchro bridge to measure odd values of θ. A one part-per-million (PPM) decade ratio-transformer (Ratio Standard), for example, is sufficiently accurate to measure the accuracy of a 2-second Synchro Standard. However, it is usually not necessary to use equation (4) to determine the error, as will be shown in the following paragraphs.

Input Impedance Unbalance

It should be pointed out at this time, that this single-arm synchro bridge technique (which consists of substituting a decade RatioTran in place of the tapped arm, and eliminating the two fixed arms) is limited to very low values of Z_{SS}. The following equation, which is derived from equations (9) and (12), gives the maximum equivalent angular error introduced by this technique.

$$\Delta\theta_{MAX\,(sec)} \cong \text{Real Part Of}\left(1.2 \times 10^5 \frac{Z_{SS}}{Z_{IN}}\right), \qquad (5)$$

where Z_{IN} is the input impedance of the ratio-transformer, which is approximately 500K for a 0.35f RatioTran at 400 cps. $\Delta\theta$ will be maximum and positive at 0°, zero at 30°, and maximum and negative at 60°.

TAP RATIO DEVIATIONS

To find the equivalent angular error of a bridge, caused by a tap ratio deviation at a particular angle, equation (2) is differentiated with respect to θ:

$$\frac{dA}{d\theta} = \frac{\sqrt{3}}{2}\csc^2(\theta+60°). \qquad (6)$$

For small changes ($\Delta\theta = 5°$ gives a maximum ΔA error of $+4.8\%$ at $\theta = 0°$ or 60°, and -2.2% at $\theta = 30°$):

$$\frac{\Delta A}{\Delta\theta} = \frac{\sqrt{3}}{2}\csc^2(\theta+60°), \qquad (7)$$

where $\Delta\theta$ is in radians. However, 4.85×10^{-6} radians is equal to 1 second. Therefore,

$$\frac{\Delta A}{\Delta\theta_{(seconds)}} = 4.85 \times 10^{-6} \times \frac{\sqrt{3}}{2}\csc^2(\theta+60°), \qquad (8)$$

or

$$\frac{\Delta A_{(ppm)}}{\Delta\theta_{(seconds)}} = 4.20\csc^2(\theta+60°). \qquad (9)$$

Figure 4, and table III are obtained from equation (9).

TABLE III

Values of $\frac{\Delta A_{(ppm)}}{\Delta\theta_{(seconds)}}$ for a Synchro Bridge in 5° Increments

Synchro Bridge Angle, θ (Degrees)	$\frac{\Delta A_{(ppm)}}{\Delta\theta_{(seconds)}}$
0, 60	5.60
5, 55	5.12
10, 50	4.76
15, 45	4.50
20, 40	4.33
25, 35	4.23
30	4.20

INPUT IMPEDANCE UNBALANCE

The equivalent angular error caused by an input impedance unbalance may be calculated by solving for the amount the ratio (A) effectively changes due to this unbalance.

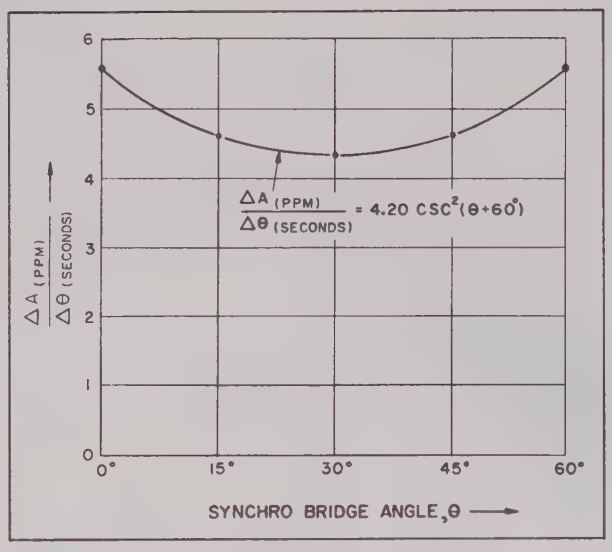

FIGURE 4. $\frac{\Delta A_{(PPM)}}{\Delta\theta_{(SECONDS)}}$ as a Function of Synchro Bridge Angle, θ.

The ratio at which a null actually exists is equal to the ratio of the two appropriate voltages at the input terminals of the bridge. Therefore, from 0° to 60°:

$$A' \equiv \text{Real Part Of} \left(\frac{V_{S3-S1}}{-V_{S1-S2}} \right). \qquad (10)$$

By loop analysis, it may be shown that:

$$A' = \text{Real Part Of} \frac{\left[\frac{2Z_{ss}}{3}\left(Z_{S2-S3} - Z_{S1-S2}\right) \sin(\theta+60^\circ) - \left(Z_{S1-S2}Z_{S2-S3} + \frac{4Z_{ss}Z_{S2-S3}}{3} + \frac{2Z_{ss}Z_{S1-S2}}{3} \right) \sin\theta \right] Z_{S3-S1}}{\left[\frac{2Z_{ss}}{3}\left(Z_{S2-S3} - Z_{S3-S1}\right) \sin\theta - \left(Z_{S3-S1}Z_{S2-S3} + \frac{4Z_{ss}Z_{S2-S3}}{3} + \frac{2Z_{ss}Z_{S3-S1}}{3} \right) \sin(\theta+60^\circ) \right] Z_{S1-S2}} . \qquad (11)$$

If the calculated value of A′ is greater than the theoretical value of A at a given bridge angle, then a negative equivalent angular error results. Therefore, by definition:

$$\Delta A' \equiv A_{(\text{theoretical})} - A'_{(\text{calculated})}, \qquad (12)$$

where $A_{(\text{theoretical})}$ may be found from equation (1). $\Delta A'$ is then converted to an equivalent angular error from equation (9).

Since most synchro bridges actually consist of only one tapped arm plus two fixed arms, the arm impedance seen by each pair of stator leads is switched every 60°. Consequently, instead of obtaining a two cycle sinusoidal error curve, the error found from equation (11) will repeat every 60° and will consist of 1/3 of a complete sine wave.

Table IV tabulates the results of equations (11), and (12) from 0° to 60°, for various combinations of input impedance unbalance. Note that variations in input impedance unbalance conditions will change the phase and/or the amplitude of the error curve. It should also be noted, that the arm with a value of impedance that is in between the other two arms has very little (13.4%) effect on the maximum amplitude of $\Delta\theta$.

TABLE IV

Equivalent Angular Error, From 0° to 60°, for Various Combinations of Synchro Bridge Input Impedance Unbalance ($Z_{ss} = 37.5\ \angle 0^\circ$ ohms)

DELTA ARM IMPEDANCE (K ohms $\angle 0^\circ$)			EQUIVALENT ANGULAR ERROR, $\Delta\theta$ (SECONDS)				
Z_{S3-S1}	Z_{S1-S2}	Z_{S2-S3}	0°	15°	30°	45°	*60°
500	450	450	0	−0.575	−1.0	−1.15	−1.0
500	450	475	+0.5	0	−0.5	−0.866	−1.0
500	450	500	+1.0	+0.575	0	−0.575	−1.0
475	450	500	+1.0	+0.866	+0.5	0	−0.5
450	450	500	+1.0	+1.15	+1.0	+0.575	0
450	475	500	+0.5	+0.866	+1.0	+0.866	+0.5
450	500	500	0	+0.575	+1.0	+1.15	+1.0
450	500	475	−0.5	0	+0.5	+0.866	+1.0
450	500	450	−1.0	−0.575	0	+0.575	+1.0
475	500	450	−1.0	−0.866	−0.5	0	+0.5
500	500	450	−1.0	−1.15	−1.0	−0.575	0
500	475	450	−0.5	−0.866	−1.0	−0.866	−0.5

* The error indicated at 60° will only be approached in actual practice, since the arms of the delta are switched every 60°.

The maximum possible error for a given set of conditions may be calculated from the following equation:

$$\Delta\theta_{MAX\ (sec)} \cong \text{Real Part Of} \left(\frac{4Z_{ss}(\Omega) \cdot Z'_{UNBALANCE}(\%)}{3Z'\ (K\Omega)} \right), \qquad (13)$$

where Z′ is the average value of $Z_{ARM\ (max)}$ and $Z_{ARM\ (min)}$, and $Z'_{UNBALANCE}$ is equal to the difference between $Z_{ARM\ (max)}$ and $Z_{ARM\ (min)}$ divided by Z′.

TOTAL EQUIVALENT ANGULAR ERROR

Thus, the total equivalent angular error of a synchro bridge at a given position is equal to the algebraic sum of the angular error due to a tap ratio deviation (ΔA), and the angular error due to an input impedance unbalance ($\Delta A'$). In terms of an equation then, obviously:

$$\Delta A = A_{(\text{measured})} - A_{(\text{theoretical})}. \qquad (14)$$

Combining this with equation (12), yields:

$$\Delta A_{total} = \Delta A + \Delta A' = A_{(\text{measured})} - A'_{(\text{calculated})}. \qquad (15)$$

Equation (9) is then used to convert the total ΔA to a total $\Delta\theta$.

It should also be mentioned that a positive angular error for a synchro (or resolver) bridge is somewhat comparable to a control transformer with a negative angular error, since for a bridge:

$$\Delta\Theta_{total} \equiv \Theta_{elect} - \Theta_{dial}. \quad (16)$$

The error of a control transformer is normally defined as:

$$\Delta\Theta_{total} \equiv \Theta_{shaft} - \Theta_{elect}, \quad (17)$$

where Θ_{elect} is defined as the electrical input angle required to produce a null at the output.

ERROR VOLTAGE GRADIENT

Assuming that the input impedance of the null detector is very much greater than the output impedance of the bridge (maximum output impedance equals 2.5K at 30° for the uncompensated 10K resistive type bridge), it may be seen from figures 1 and 2 that:

$$V_{R2-R1} = -\left[\sin\Theta + A\sin(\Theta+240°)\right] E_{s(max)}, \quad (18)$$

where $E_{s(max)}$ is the maximum RMS stator output voltage of the synchro under test, and V_{R2-R1} is out-of-phase with E_{R2-R1} of the synchro under test for a positive angular error (Θ_{elect} greater than Θ_{shaft}).

To find the equivalent angular error of the output error voltage from a bridge at a particular angle, equation (18) is differentiated with respect to Θ:

$$\frac{dV_{R2-R1}}{d\Theta} = -\left[\cos\Theta + A\cos(\Theta+240°)\right] E_{s(max)}. \quad (19)$$

Substituting equation (1) for A, disregarding the negative sign, and for fairly large changes ($\Delta\Theta = 5°$, gives a ΔV error of approximately +0.13% at any angle Θ):

$$\frac{\Delta V_{R2-R1}}{\Delta\Theta} = \left[\cos\Theta - \sin\Theta\cot(\Theta+60°)\right] E_{s(max)}, \quad (20)$$

where $\Delta\Theta$ is in radians, and Θ is the angle of the bridge.

Converting $\Delta\Theta$ from radians to seconds, and letting

$$K \equiv \left[\cos\Theta - \sin\Theta\cot(\Theta+60°)\right] \quad (21)$$

yields:

$$\frac{\Delta V_{R2-R1\ (volts)}}{\Delta\Theta_{(seconds)}} = 4.85 \times 10^{-6}\, K\, E_{s(max)}. \quad (22)$$

Figure 5 and table V show the relationship between K and Θ; therefore, demonstrating how $\frac{\Delta V_{R2-R1}}{\Delta\Theta}$ varies with Θ.

TABLE V

Values of K for a Synchro Bridge in 5° Increments

Synchro Bridge Angle, Θ (Degrees)	K
0, 60	1.000
5, 55	0.956
10, 50	0.922
15, 45	0.897
20, 40	0.880
25, 35	0.869
30	0.866

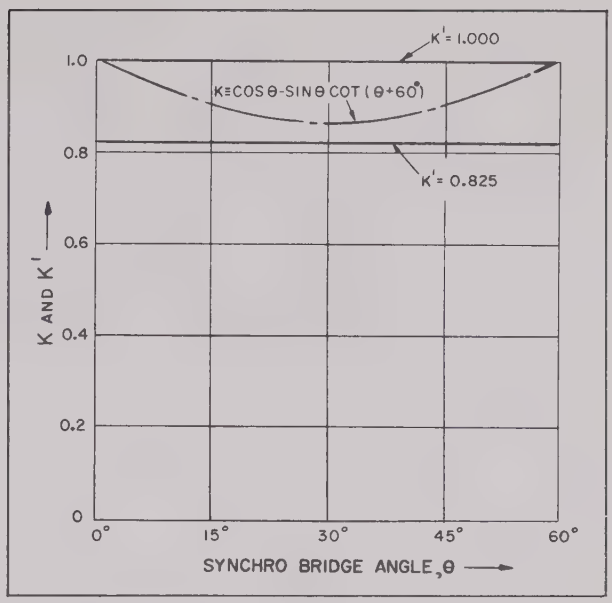

FIGURE 5. K, and K′ as a Function of Synchro Bridge Angle, Θ.

VOLTAGE GRADIENT COMPENSATING NETWORK

A voltage gradient compensating network may be added to the RatioTran Synchro Bridge illustrated in figure 2, which will adjust K to the same value at each angle and, therefore, make $\frac{\Delta V_{R2-R1}}{\Delta\Theta}$ independent of Θ. This type of delta bridge (with a voltage gradient compensating network) is used with what is referred to as the "Proportional Voltage Gradient Method" of testing by MIL-S-20708A.

Basically, K may be adjusted to the same value at each angle with the use of a scaling transformer that has a special tap for each 5° position of the bridge. A compensated RatioTran Synchro Bridge is illustrated in figure 6, which shows the individual taps for each position.

Usually, it is desirable to adjust K to 1.000 (K′) at each position as shown in figure 5, in order to simplify equation (22). This is accomplished by locating the taps so that the transformation ratio of the scaling transformer at each angle is:

$$T.R. = \frac{K'}{K} = \frac{1.000}{K}. \quad (23)$$

A compensated synchro bridge with a K′ equal to 1.000 is equivalent to a synchro control transformer (CT) with a transformation ratio equal to 1.155*.

1.000 may now be substituted for K in equation (22); therefore, $\frac{\Delta V_{R2-R1}}{\Delta\Theta}$ becomes:

$$\frac{\Delta V_{R2-R1\ (volts)}}{\Delta\Theta_{(seconds)}} = 4.85 \times 10^{-6}\, E_{s(max)}. \quad (24)$$

* The transformation ratio of a CT, as defined by MIL-S-20708A, is equivalent to the rotor voltage at maximum coupling divided by $0.866\ E_{s(max)}$.

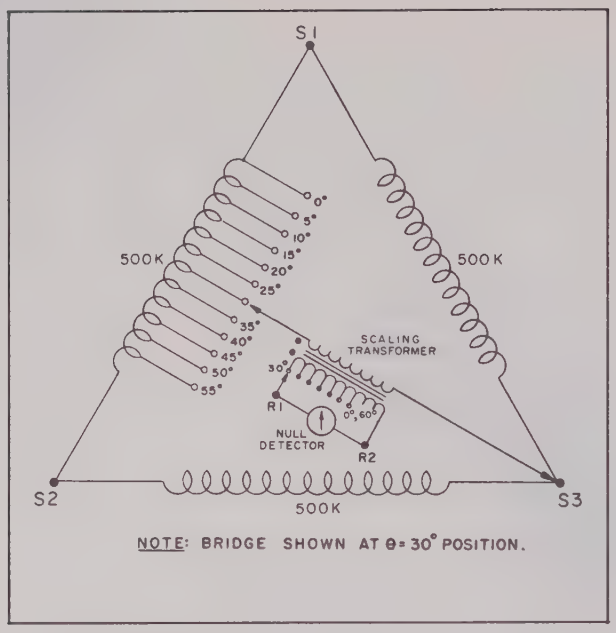

FIGURE 6. Compensated RatioTran Synchro Bridge.

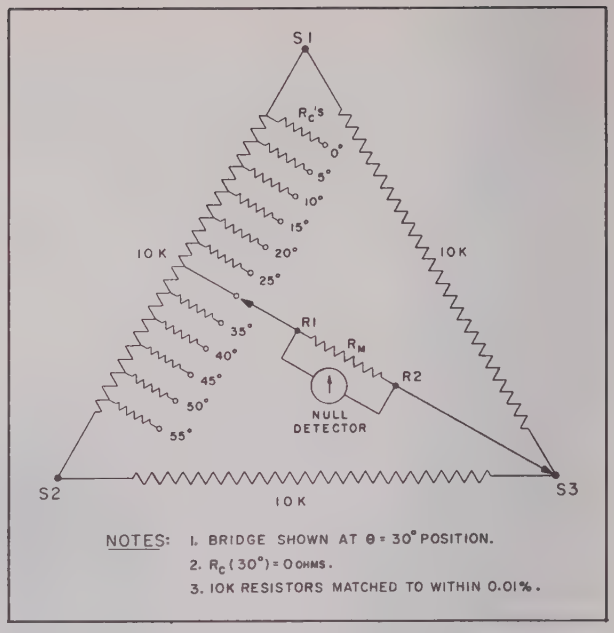

FIGURE 7. Compensated Resistive Synchro Bridge.

RESISTIVE SYNCHRO BRIDGE

In general, all of the equations derived for the RatioTran Synchro Bridge are also applicable to the resistive synchro bridge. However, resistive synchro bridges are usually compensated with resistors, instead of a scaling transformer.

VOLTAGE GRADIENT COMPENSATING NETWORK

Resistive synchro bridges may be compensated by using a separate resistive voltage divider at each angular position of the bridge, as shown in figure 7.

Since $K_{minimum}$ is equal to 0.866, it is desirable to attenuate K to 0.866 at each position in order to obtain maximum null sensitivity. It is because of this, that R_C at Θ equals 30° is zero.

From figures 5 and 7, it may appear that K can be attenuated to 0.866 at each angle by letting:

$$\left(\frac{R_M}{R_M+R_C}\right)K=0.866. \qquad (25)$$

However, due to the inherently high output impedance of the resistive bridge, this equation is only valid when the output impedance is negligible with respect to R_M. The maximum deviation from 0.866 will occur at 30°, where the output impedance is maximum (2.5K).

The output impedance of the resistive bridge (R_O) is taken into account, by considering it to be effectively in series with R_C. K′ may then be defined as the actual value to which K can be attenuated to, and held constant.

$$K'=\left(\frac{R_M}{R_M+R_C+R_O}\right)K, \qquad (26)$$

where:

$$R_O=A\,(1-A)\,10K \qquad (27)$$

for an uncompensated 10K resistive synchro bridge.

Since the values of K, R_C (30°), and R_O are fixed, it is clear from equation (26) that the value selected for R_M will determine the value of K′. As mentioned before, the larger R_M is with respect to R_O, the closer K′ will be to 0.866 and the higher the null sensitivity will be. However, if R_M is made too large, the input impedance of the null detector becomes an undesirable significant factor.

A good compromise between maximum null sensitivity, and independence of null detector input impedance for a resistive bridge is to let R_M equal 50K. K′ is then equal to 0.825 at each angle, as shown in figure 5. R_C at 0° then becomes approximately 10.6K ohms. However, the output impedance is now approximately 8.8K at 0°, and 2.4K at 30°.

0.825 may now be substituted for K in equation (22); therefore, $\frac{\Delta V_{R2-R1}}{\Delta\Theta}$ becomes:

$$\frac{\Delta V_{R2-R1(volts)}}{\Delta\Theta\ (seconds)}=4\times10^{-6}\,E_{s(max)}. \qquad (28)$$

NOTE: IN PREVIOUS EDITIONS OF THIS ANALYSIS, THE Z_{SS} (OUTPUT IMPEDANCE) OF A CX WAS ASSUMED TO BE THE TOTAL IMPEDANCE MEASURED ACROSS TWO OF THE OUTPUT (STATOR) TERMINALS WITH THE THIRD OUTPUT TERMINAL OPEN AND THE ROTOR SHORTED. IN THIS EDITION, EQUATIONS (5), (11), AND (13) PLUS TABLE IV HAVE BEEN REVISED TO AGREE WITH THE DEFINITION IN THE B REVISION OF MIL-S-20708. THIS "MIL-SPEC" DEFINITION STATES THAT Z_{SS} SHALL BE MEASURED ACROSS TWO OUTPUT TERMINALS, WITH THE THIRD TERMINAL CONNECTED TO ONE OF THE OTHER TWO AND THE ROTOR SHORTED. THIS MEANS, FOR EXAMPLE, THAT A CX WHICH WAS FORMERLY SAID TO HAVE A Z_{SS} OF 100 OHMS, IS NOW SAID TO HAVE A Z_{SS} OF 75 OHMS.

PART II
RESOLVER BRIDGES

RESISTIVE 4-ARM RESOLVER BRIDGE

A resistive 4-arm resolver bridge is illustrated in figure 8. This type of bridge (without a voltage gradient compensating network) is used with what is referred to as the "Proportional Voltage Nulling Method" of testing by MIL-R-23417.

Figure 9 illustrates the phase relationships of the output voltages of a standard data-transmission type resolver control transmitter (RX), which are normally applied to the four input terminals of a resolver bridge. These phase relationships are derived from the standard resolver equations for rotor energized resolvers (RX's):

$$E_{S1-S3} = N\,(E_{R1-R3}\cos\Theta - E_{R2-R4}\sin\Theta) \qquad (29)$$

$$E_{S2-S4} = N\,(E_{R2-R4}\cos\Theta + E_{R1-R3}\sin\Theta) \qquad (30)$$

For stator energized resolvers (RD's, and RC's) the equations become:

$$E_{R1-R3} = N\,(E_{S1-S3}\cos\Theta + E_{S2-S4}\sin\Theta) \qquad (31)$$

$$E_{R2-R4} = N\,(E_{S2-S4}\cos\Theta - E_{S1-S3}\sin\Theta) \qquad (32)$$

In the above equations, N is equal to the transformation ratio from primary to secondary, and Θ is positive for CCW rotation from electrical zero (EZ) when viewed from the shaft end.

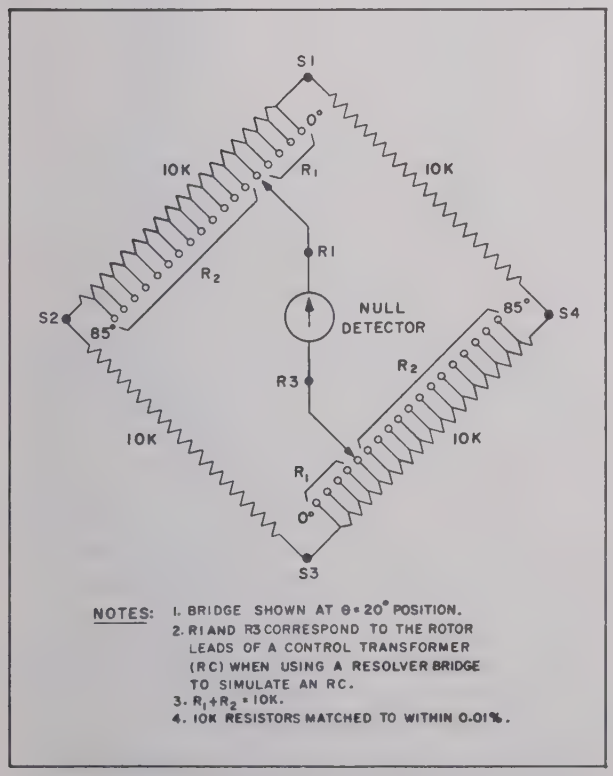

FIGURE 8. Resistive 4-Arm Resolver Bridge

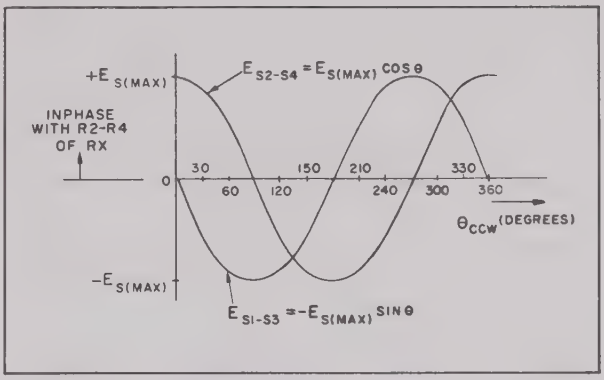

FIGURE 9. Standard Resolver Control Transmitter (RX) Outputs as a Function of CCW Rotation From Electrical Zero (EZ), With R2−R4 Excited.

IDEAL TAP RATIOS

At null, when Θ is equal to any angle between 0° and 90°, it can be proven that, ideally, for the position of the tapped arms shown:

$$\frac{R_1}{R_2} = \tan\Theta, \qquad (33)$$

which is the equation usually found in resolver specifications. Table VI lists the positions of the elements within a 4-arm resolver bridge from 0° to 360°.

TABLE VI

Positions of Elements Within a 4-Arm Resolver Bridge From 0° to 360°

Resolver Bridge Angle, Θ (Degrees)	Position of Tapped Arms (R_1 to R_2)	
	A_{R1}	A_{R3}
0-90	S1-S2	S3-S4
90-180	S2-S3	S4-S1
180-270	S3-S4	S1-S2
270-360	S4-S1	S2-S3

In the following analysis, Θ is generally restricted to angles between 0° and 90°. However, since the bridge characteristics repeat every 90°, values of Θ not within this range may be converted by subtracting a multiple of 90°.

With the use of trigonometric identities and equation (33), it can be shown that:

$$\frac{R_1}{R_1+R_2} = \tfrac{1}{2} + \tfrac{1}{2}\tan(\Theta - 45°). \qquad (34)$$

Since the resolver bridge has a pair of tapped arms which follow the same equation, the average ratio of the two arms must be considered in practice. Let $A_{R1} \equiv \frac{R_1}{R_1+R_2}$ for the arm associated with the R1 terminal of the null detector, and $A_{R3} \equiv \frac{R_1}{R_1+R_2}$ for the arm associated with the R3 terminal of the null detector. Therefore, the equivalent A is equal to:

$$A = \frac{A_{R1}+A_{R3}}{2}. \quad (35)$$

Equation (34) may then be written as:

$$A = \tfrac{1}{2} + \tfrac{1}{2} \tan (\theta - 45°). \quad (36)$$

A plot of this equation as a function of θ yields a curve similar to figure 3; however, the terminal linearity of the tapped arm is approximately 4.5%. Therefore, an equivalent maximum angular error of 4° (0.045 x 90° = 4°) would result, if the taps were equally spaced.

The precise values of A, for 5° increments are listed in table VII.

TABLE VII

Precise Values of A for a 4-Arm Resolver Bridge in 5° Increments

Resolver Bridge Angle, θ (Degrees)	$A = \frac{A_{R1}+A_{R3}}{2}$
0	.0000000
5	.0804502
10	.1498962
15	.2113249
20	.2668462
25	.3180149
30	.3660254
35	.4118364
40	.4562556
45	.5000000
50	.5437444
55	.5881636
60	.6339746
65	.6819851
70	.7331538
75	.7886751
80	.8501038
85	.9195498
90	1.0000000

Note, that at any angle:

$$A_{(\theta)} + A_{(90° - \theta)} = 1.0000000. \quad (37)$$

Equation (36) may be solved for θ, which results in:

$$\theta = \tan^{-1} \frac{A}{1-A}. \quad (38)$$

TAP RATIO DEVIATIONS

In order to find the equivalent angular error caused by an average tap ratio deviation at a particular angle for a 4-arm resolver bridge, equation (36) is differentiated with respect to θ:

$$\frac{dA}{d\theta} = \tfrac{1}{2} \sec^2 (\theta - 45°). \quad (39)$$

For small changes ($\Delta\theta = 5°$ gives a maximum ΔA error of +8.5% at $\theta = 0°$ or 90°, and −0.22% at $\theta = 45°$), and converting $\Delta\theta$ from radians to seconds and ΔA to parts-per-million yields:

$$\frac{\Delta A_{(ppm)}}{\Delta \theta_{(seconds)}} = 2.425 \sec^2 (\theta - 45°). \quad (40)$$

A plot of this equation as a function of θ yields a curve similar to figure 4. Table VIII lists the values of $\frac{\Delta A}{\Delta \theta}$ for 5° increments.

TABLE VIII

Values of $\frac{\Delta A_{(ppm)}}{\Delta \theta_{(seconds)}}$ for a 4-Arm Resolver Bridge in 5° Increments

Resolver Bridge Angle, θ (Degrees)	$\frac{\Delta A_{(ppm)}}{\Delta \theta_{(seconds)}}$
0, 90	4.85
5, 85	4.13
10, 80	3.62
15, 75	3.26
20, 70	2.96
25, 65	2.75
30, 60	2.60
35, 55	2.50
40, 50	2.45
45	2.425

ERROR VOLTAGE GRADIENT

Assuming that the input impedance of the null detector is very much greater than the output impedance of the bridge (maximum output impedance equals 5K at 45° for the uncompensated 10K resistive 4-arm bridge), it may be shown by loop analysis that:

$$V_{R3-R1} = [(1-A) \sin\theta - A\cos\theta] E_{s(max)}, \quad (41)$$

where $E_{s(max)}$ is the maximum RMS stator output voltage of the resolver under test, and V_{R3-R1} is inphase with E_{R2-R4} of the resolver under test for a positive angular error (θ_{elect} greater than θ_{shaft}).

To find the equivalent angular error of the output error voltage from a bridge at a particular angle, equation (41) is differentiated with respect to θ:

$$\frac{dV_{R3-R1}}{d\theta} = [(1-A) \cos\theta + A \sin\theta] E_{s(max)}. \quad (42)$$

Substituting $\frac{\tan\Theta}{1+\tan\Theta}$ for A, and for fairly large changes ($\Delta\Theta=5°$ gives a ΔV error of approximately $+0.13\%$ at any angle Θ):

$$\frac{\Delta V_{R3-R1}}{\Delta\Theta}=\frac{1}{\sqrt{2}}\sec(\Theta-45°)E_{s(max)}, \qquad (43)$$

where $\Delta\Theta$ is in radians, and Θ is the angle of the bridge.

Converting $\Delta\Theta$ from radians to seconds, and letting

$$K\equiv\frac{1}{\sqrt{2}}\sec(\Theta-45°) \qquad (44)$$

yields:

$$\frac{\Delta V_{R3-R1(volts)}}{\Delta\Theta_{(seconds)}}=4.85\times10^{-6}\,K\,E_{s(max)}. \qquad (45)$$

A plot of K as a function of Θ yields a curve similar to figure 5. Table IX lists the values of K for 5° increments.

TABLE IX

Values of K for a 4-Arm Resolver Bridge in 5° Increments

Resolver Bridge Angle, Θ (Degrees)	K
0, 90	1.000
5, 85	.924
10, 80	.864
15, 75	.817
20, 70	.780
25, 65	.750
30, 60	.732
35, 55	.718
40, 50	.710
45	.707

VOLTAGE GRADIENT COMPENSATING NETWORK

As was the case for the resistive synchro bridge, a voltage gradient compensating network may be added to the resistive 4-arm resolver bridge which will attenuate K to a constant and, therefore, make $\frac{\Delta V}{\Delta\Theta}$ independent of Θ. MIL-R-23417 refers to the technique which utilizes this type of bridge (with a voltage gradient compensating network) as the "Proportional Voltage Gradient Method".

In order to attenuate K to a constant, a separate resistive voltage divider is used at each 5° position of the bridge, as was done for the resistive synchro bridge illustrated in figure 7.

From table IX it is seen that $K_{minimum}$ for a 4-arm resolver bridge is equal to 0.707. Since this minimum value occurs at 45°, R_C at this position must be zero in order to obtain maximum null sensitivity. Unfortunately, the output impedance of the bridge is maximum (5K) at 45°; therefore, unless the output impedance (R_O) is negligible with respect to R_M, K will be attenuated an additional amount.

Equation (26) for K′ is also valid for the resistive 4-arm resolver bridge, when considering the effect of the output impedance. However, the output impedance is equal to:

$$R_O=2A\,(1-A)\,10K, \qquad (46)$$

for an uncompensated 10K resistive 4-arm resolver bridge.

A value of 50K for R_M is a good compromise between maximum null sensitivity, and independence of null detector input impedance for a resistive resolver bridge also. K′ is then equal to 0.643 at each angle, which is equivalent to a resolver control transformer (RC) with a transformation ratio equal to 0.643. R_C at 0° then becomes approximately 27.8K ohms. However, the output impedance is now approximately 18K at 0°, and 4.6K at 45°.

0.643 may now be substituted for K in equation (45); therefore, $\frac{\Delta V_{R3-R1}}{\Delta\Theta}$ becomes:

$$\frac{\Delta V_{R3-R1\ (volts)}}{\Delta\Theta_{\ (seconds)}}=3.12\times10^{-6}\,E_{s(max)}. \qquad (47)$$

IMPEDANCE UNBALANCE

Errors due to impedance unbalance in the 4-arm resolver bridge may be separated into two types. First there will be an error due to an input impedance unbalance, and second there will be an additional error if the forward and reverse transfer functions are not zero. In order for both of these errors to be zero, all four arms of the bridge must have the same impedance.

Input Impedance Unbalance

The error due to an unbalanced input impedance may be derived as follows. Consider the load on the sine output of the resolver to be Z_{S3-S1}, and the load on the cosine output to be Z_{S2-S4}. For an ideal resolver:

$$\frac{E_{S3-S1}}{E_{S2-S4}}=\tan\Theta. \qquad (48)$$

Letting $A\equiv\frac{E_{S3-S1}}{E_{S2-S4}}$ from 0° to 90°, differentiating equation (48), and converting from radians to seconds yields:

$$\frac{\Delta A}{\Delta\Theta_{(seconds)}}=4.85\times10^{-6}\sec^2\Theta. \qquad (49)$$

The voltage ratio at the input terminals of the bridge is defined as:

$$A'\equiv\text{Real Part Of}\left(\frac{V_{S3-S1}}{V_{S2-S4}}\right), \qquad (50)$$

where:

$$V_{S3-S1}=\left(\frac{Z_{S3-S1}}{Z_{S3-S1}+Z_{SS}}\right)E_{S3-S1} \qquad (51)$$

$$V_{S2-S4}=\left(\frac{Z_{S2-S4}}{Z_{S2-S4}+Z_{SS}}\right)E_{S2-S4}, \qquad (52)$$

and

$$\Delta A'\equiv A_{(theoretical)}-A'_{(calculated)}, \qquad (53)$$

which is identical to equation (12).

Therefore, from equations (48), (50), and (53): $\Delta A'\cong$ Real Part Of

$$\left(\frac{Z_{S2-S4}}{Z_{S2-S4}+Z_{SS}}-\frac{Z_{S3-S1}}{Z_{S3-S1}+Z_{SS}}\right)\tan\Theta. \qquad (54)$$

Combining equations (49) and (54):

$\Delta\Theta_{(\text{seconds})}$ = Real Part Of

$$1.03 \times 10^5 \left(\frac{Z_{S2-S4}}{Z_{S2-S4}+Z_{SS}} - \frac{Z_{S3-S1}}{Z_{S3-S1}+Z_{SS}} \right) \sin 2\Theta. \quad (55)$$

This equation is only valid from 0° to 90°, since most 4-arm resolver bridges consist of two tapped arms plus two fixed arms. Therefore, the error found from 0° to 90° will repeat every 90°. Equation (56), which is derived from equation (55), may be used to find the maximum error for a given set of conditions:

$\Delta\Theta_{MAX\ (sec)} \cong$ Real Part Of

$$\left(\frac{Z_{SS}(\Omega) \cdot Z'_{UNBALANCE}\ (\%)}{Z'\ (K\Omega)} \right), \quad (56)$$

where Z′ is the average value of Z_{S3-S1} and Z_{S2-S4}, and $Z'_{UNBALANCE}$ is equal to the difference between Z_{S3-S1} and Z_{S2-S4} divided by Z′.

Note from figure 8, that Z_{S3-S1} will equal Z_{S2-S4} when either one or both of the pairs of opposite arms are equal, i.e. $Z_{S1-S2}=Z_{S4-S3}$ and/or $Z_{S2-S3}=Z_{S1-S4}$.

Non-Zero Transfer Functions

Assuming $Z_{S3-S1}=Z_{S2-S4}$, and for simplicity both $Z_{S1-S2}=Z_{S4-S3}=Z_T$ ($V_{S1-S2}=V_{S4-S3}$) and $Z_{S2-S3}=Z_{S1-S4}=Z_F$ ($V_{S2-S3}=V_{S1-S4}$), the equivalent angular error due to a non-zero ($Z_T \neq Z_F$) forward (S3−S1 to S2−S4) and reverse (S2−S4 to S3−S1) transfer function may be calculated.

By setting V_{R3-R1} equal to zero and solving for the ratio at which a null actually exists, it may be shown that:

A′ = Real Part Of

$$\left(\tfrac{1}{2} + \frac{Z_F\ [Z_{SS}+Z_T(Z_T+Z_F+Z_{SS})]\ (\sin\Theta-\cos\Theta)}{2Z_T\ [Z_{SS}+Z_F(Z_T+Z_F+Z_{SS})]\ (\sin\Theta+\cos\Theta)} \right). \quad (57)$$

The equivalent angular error at each angle may then be calculated from equations (36), (40), and (53). The resulting error from 0° to 90° will be a $\pm(\Delta\Theta_{MAX})\cos 2\Theta$ function, which will repeat every 90°. Equation (56) may be used to find $\Delta\Theta_{MAX}$, by using Z_T and Z_F in place of Z_{S3-S1} and Z_{S2-S4}.

Note from figure 8, that the forward and reverse transfer function will be equal if Z_{S3-S1} equals Z_{S2-S4}, and will be zero if $Z_{S1-S2}=Z_{S2-S3}$ and $Z_{S1-S4}=Z_{S4-S3}$, or if $Z_{S1-S2}=Z_{S1-S4}$ and $Z_{S2-S3}=Z_{S4-S3}$.

TOTAL EQUIVALENT ANGULAR ERROR

The total equivalent angular error of a resolver bridge at a given position is equal to the algebraic sum of the angular error due to a tap ratio deviation ($\triangle$A), and the angular error due to an impedance unbalance ($\triangle$A′). The error due to an impedance unbalance as mentioned before, may be due to either one or both of two causes. The error due to an unbalanced input impedance, the first cause, should be calculated directly from equation (55). The error due to a non-zero forward and reverse transfer function (peculiar only to the 4-arm resolver bridge), the second cause, may be calculated from equation (57) for A′. Recall though, that equation (57) is based on the assumption that $Z_{S1-S2}=Z_{S4-S3}$, and $Z_{S2-S3}=Z_{S1-S4}$.

In terms of an equation then:

$$\Delta\Theta_{total} = \Delta\Theta_{(\text{tap ratio deviation})} + \Delta\Theta_{(\text{input Z unbalance})} + \Delta\Theta_{(\text{non-zero transfer function})}, \quad (58)$$

for all resolver bridges.

A characteristic of the 4-arm resistive resolver bridge which is somewhat questionable as far as its equivalent angular error is concerned, is the fact that the center taps of the two output windings of the resolver under test are effectively tied together through a 2.5K resistor, assuming that Z_{SS} is negligible.

RATIOTRAN RESOLVER BRIDGE

A RatioTran Resolver Bridge is illustrated in figure 10, which provides complete isolation to the two output windings of the resolver under test. In addition, it will be noted that the forward and reverse transfer function is inherently zero.

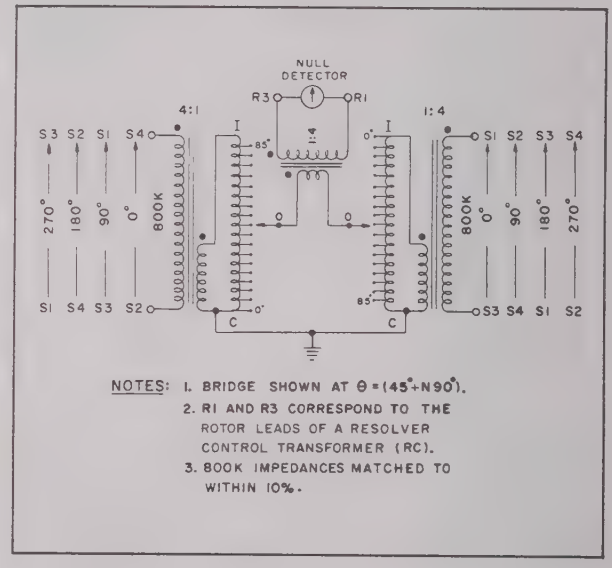

FIGURE 10. RatioTran Resolver Bridge.

IDEAL TAP RATIOS

At null, when Θ is between 0° and 90°:

$$E_{S2-S4} \cdot A_{R3} = E_{S3-S1} \cdot A_{R1}, \quad (59)$$

where $A \equiv \frac{OC}{IC}$, and A_{R3} and A_{R1} are the ratios of the two arms associated with the null detector terminals R3 and R1 respectively.

Since E_{S2-S4} and E_{S3-S1} are cosine and sine functions respectively, it is clear that the ratio of A_{R3} to A_{R1} must equal the tangent of Θ. One solution would be to let A_{R3} equal A and A_{R1} equal (1−A), where A is defined in equation (36). In which case, all of the equations derived for the 4-arm bridge are applicable. However, a more

straightforward and advantageous solution is to let A_{R3} and A_{R1} equal the sine and cosine respectively:

$$A_{R3} = \sin \Theta \quad (60)$$

$$A_{R1} = \cos \Theta. \quad (61)$$

Therefore,

$$\frac{A_{R3}}{A_{R1}} = \tan \Theta. \quad (62)$$

The precise values of A_{R3} and A_{R1}, for 5° increments, are listed in table X.

TABLE X

Precise Values of A_{R3} and A_{R1} for a RatioTran Resolver Bridge in 5° Increments

Resolver Bridge Angle, Θ (Degrees)	$A_{R3} \equiv \frac{OC}{IC}$	$A_{R1} \equiv \frac{OC}{IC}$
0	.0000000	1.0000000
5	.0871557	.9961947
10	.1736482	.9848078
15	.2588190	.9659258
20	.3420201	.9396926
25	.4226183	.9063078
30	.5000000	.8660254
35	.5735764	.8191520
40	.6427876	.7660444
45	.7071068	.7071068
50	.7660444	.6427876
55	.8191520	.5735764
60	.8660254	.5000000
65	.9063078	.4226183
70	.9396926	.3420201
75	.9659258	.2588190
80	.9848078	.1736482
85	.9961947	.0871557
90	1.0000000	.0000000

TAP RATIO DEVIATIONS

To find the equivalent angular error caused by a tap ratio deviation by both A_{R3} and A_{R1}, equation (62) must first be solved for Θ:

$$\Theta = \tan^{-1} \frac{A_{R3}}{A_{R1}}. \quad (63)$$

Now, since both A_{R3} and A_{R1} are variables, equation (63) must be differentiated by parts, which yields:

$$\left| \frac{\partial \Theta_{R3}}{\partial A_{R3}} \right|_{A_{R1} = \text{constant}} = \cos \Theta, \quad (64)$$

and

$$\left| \frac{\partial \Theta_{R1}}{\partial A_{R1}} \right|_{A_{R3} = \text{constant}} = -\sin \Theta. \quad (65)$$

However, for small changes:

$$\Delta\Theta_{TOTAL} = \Delta\Theta_{R3} + \Delta\Theta_{R1}. \quad (66)$$

Therefore, combining equations (64), (65), and (66), and converting $\Delta\Theta$ from radians to seconds and ΔA to parts-per-million yields:

$$\Delta\Theta_{TOTAL\ (seconds)} = 0.206\ (\Delta A_{R3\ (ppm)} \cos \Theta - \Delta A_{R1\ (ppm)} \sin \Theta), \quad (67)$$

where ΔA is positive if the measured value of A is larger than the theoretical value.

ERROR VOLTAGE GRADIENT

From figure 10, it may be seen that:

$$V_{R3-R1} = (A_{R1} \sin \Theta - A_{R3} \cos \Theta) E_{s(max)}. \quad (68)$$

To find the equivalent angular error of the output error voltage from a bridge at a particular angle, equation (68) is differentiated with respect to Θ:

$$\frac{dV_{R3-R1}}{d\Theta} = (A_{R1} \cos \Theta + A_{R3} \sin \Theta) E_{s(max)}. \quad (69)$$

Substituting $\cos \Theta$ and $\sin \Theta$ for A_{R1} and A_{R3} respectively, and for fairly large changes ($\Delta\Theta = 5°$ gives a ΔV error of approximately $+0.13\%$ at any angle Θ):

$$\frac{\Delta V_{R3-R1}}{\Delta\Theta} = E_{s(max)}, \quad (70)$$

where $\Delta\Theta$ is in radians. Note, that $\frac{\Delta V}{\Delta\Theta}$ is independent of the angular setting of the bridge; therefore, a voltage gradient compensating network is not required. This is the advantage of letting A_{R3} and A_{R1} equal the sine and cosine respectively. Note also, that the transformation ratio is unity.

Converting $\Delta\Theta$ from radians to seconds yields:

$$\frac{\Delta V_{R3-R1\ (volts)}}{\Delta\Theta_{\ (seconds)}} = 4.85 \times 10^{-6} E_{s(max)}. \quad (71)$$

INPUT IMPEDANCE UNBALANCE

Since the forward and reverse transfer function of the RatioTran Resolver Bridge is inherently zero, the total equivalent angular error due to an impedance unbalance will be due to an input impedance unbalance. Equation (55) may be used to calculate this error, and equation (58) may be used to find the total equivalent angular error.

The purpose of the 4:1 step down ratio for the input isolation transformers is to reduce the reflected impedance (due to switch wiring capacitance and ratio-transformer input impedance) from secondary to primary by a factor of 16. Consequently, it is possible to obtain higher values of input impedance.

Differences in transformation ratio between the two input isolation transformers will also result in a $\pm(\Delta\Theta_{MAX})$ sin 2Θ error function; therefore, internal means are provided to adjust the equality of the transformation ratios to better than 2 PPM (equivalent to a $\Delta\Theta_{MAX}$ of 0.2 sec).

SINGLE-ARM RESOLVER BRIDGE

Oftentimes the single-arm resolver bridge illustrated in figure 11 is used to measure the accuracy of Resolver

Standards. Normally, this bridge is not suitable for testing resolvers, due to the common connection between the two output windings of the resolver under test. This common connection unbalances the inherent distributed capacitance between the output windings of a resolver, therefore causing erroneous errors proportional to Z_{SS} to occur.

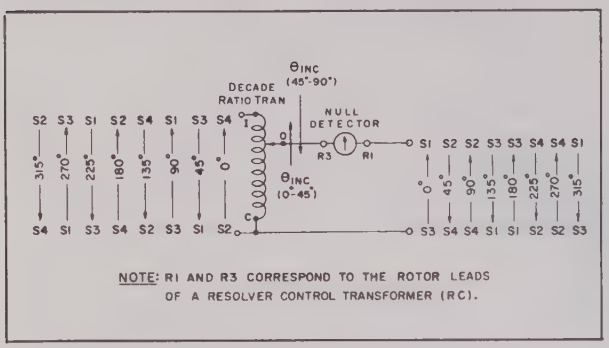
θINC (45°-90°)
DECADE RATIO TRAN
NULL DETECTOR
S2 S3 S1 S2 S4 S1 S3 S4
315° 270° 225° 180° 135° 90° 45° 0°
R3
R1
θINC (0°-45°)
S4 S1 S3 S4 S2 S3 S1 S2
S1 S2 S2 S3 S3 S4 S4 S1
0° 45° 90° 135° 180° 225° 270° 315°
S3 S4 S4 S1 S1 S2 S2 S3
NOTE: R1 AND R3 CORRESPOND TO THE ROTOR LEADS OF A RESOLVER CONTROL TRANSFORMER (RC).

FIGURE 11. Single-Arm Resolver Bridge.

IDEAL RATIOS

At null, when θ is between 0° and 45°:

$$A = \tan \theta, \tag{72}$$

where $A \equiv \frac{OC}{IC}$. The precise values of A for 5° increments are listed in table XI.

TABLE XI

Precise Values of A for a Single-Arm Resolver Bridge in 5° Increments

Resolver Bridge Angle, θ (Degrees)	$A \equiv \frac{OC}{IC}$
0, 90	.0000000
5, 85	.0874887
10, 80	.1763270
15, 75	.2679492
20, 70	.3639702
25, 65	.4663077
30, 60	.5773503
35, 55	.7002075
40, 50	.8390996
45	1.0000000

RATIO DEVIATIONS

Equation (72) is differentiated with respect to θ, in order to find the equivalent angular error represented by a deviation in ratio:

$$\frac{dA}{d\theta} = \sec^2 \theta. \tag{73}$$

For small changes, and converting $\triangle\theta$ from radians to seconds and $\triangle A$ to parts-per-million yields:

$$\frac{\triangle A_{(ppm)}}{\triangle \theta_{(seconds)}} = 4.85 \sec^2 \theta. \tag{74}$$

Table XII lists the values of $\frac{\triangle A}{\triangle \theta}$ for 5° increments.

TABLE XII

Values of $\frac{\triangle A_{(ppm)}}{\triangle \theta_{(seconds)}}$ for a Single-Arm Resolver Bridge in 5° Increments

Resolver Bridge Angle, θ (Degrees)	$\frac{\triangle A_{(ppm)}}{\triangle \theta_{(seconds)}}$
0, 90	4.85
5, 85	4.90
10, 80	5.00
15, 75	5.20
20, 70	5.50
25, 65	5.92
30, 60	6.52
35, 55	7.24
40, 50	8.26
45	9.70

ERROR VOLTAGE GRADIENT

From figure 11, it may be seen that:

$$V_{R3\text{-}R1} = (\sin \theta - A \cos \theta) E_{s(max)}. \tag{75}$$

Therefore,

$$\frac{\triangle V_{R3\text{-}R1\ (volts)}}{\triangle \theta_{(seconds)}} = 4.85 \times 10^{-6} \left(\frac{1}{\cos \theta}\right) E_{s(max)}. \tag{76}$$

INPUT IMPEDANCE UNBALANCE

Disregarding the equivalent angular error caused by the common connection, there will be an additional error due to the unbalanced input impedance. From equation (55) the absolute value of the maximum error will occur at 45°, and is equal to:

$$\triangle \theta_{MAX\ (sec)} \cong \text{Real Part Of} \left(1.03 \times 10^5 \frac{Z_{SS}}{Z_{IN}}\right), \tag{77}$$

where Z_{IN} is the input impedance of the ratio-transformer, which is approximately 500K for a 0.35f RatioTran at 400 cps. $\triangle\theta$ will be zero at 0°, maximum negative at 40°, and maximum and positive at 45°, assuming quadrant switching is performed between 40° and 45°.

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APPLICATION NOTE NO. 7

MEASURING SYNCHRO/RESOLVER TRANSFORMATION RATIO AND PHASE SHIFT, PLUS FUNDAMENTAL AND TOTAL NULL

GENERAL

Since conventional synchros and resolvers are, basically, variable coupling transformers, it follows that they will normally have a leading time phase shift (ϕ) from primary to secondary as well as a maximum transformation ratio (N) at maximum coupling. Furthermore, since they are not "ideal" variable coupling transformers, the output voltage will never be zero and can, therefore, only be minimized. The fundamental and total null voltages are measured at this minimum output voltage position.

The output voltages of a synchro (CX) at 30° (first position of maximum coupling from EZ) can be represented by the following vector diagram. The outputs of resolvers, and other types of synchros are quite similar.

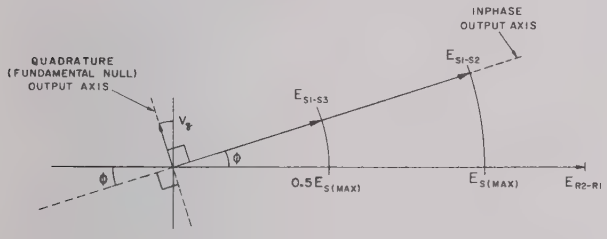

FIGURE 1. *Vector Diagram of CX Output Voltages at* $\Theta = 30°$.

DEFINITIONS

Transformation Ratio (N) is the ratio of the fundamental component of the no-load secondary voltage, at the first position of maximum coupling in the positive (CCW, as viewed from the shaft end) direction from electrical zero (EZ) to the fundamental voltage applied to the primary.

Typical values and tolerances: 0.1 to 2.3, $\pm$.1 to 4% (approximately 1.0 $\pm$.01 to .1% for some computational-type resolvers).

Phase Shift (ϕ) is the difference between the time phase of the fundamental component of the primary and secondary voltages, at the first position of maximum coupling in the positive direction from EZ.

Typical values and tolerances: 0° to + 20°, $\pm$0.5° to 4° (0° $\pm$1 to 4 minutes for some computational-type resolvers).

Fundamental Null is the fundamental component of the minimum output voltage. Fundamental null voltage is synonymous with quadrature voltage (V_Q), since the inphase output voltage at this point is zero. Referring to figure 1, note that V_Q could *not* be measured at the angle shown, because the inphase components are so much larger in amplitude.

Typical values: 15 to 90 MV

Total Null is the RMS value of the minimum output voltage, including harmonics.

Typical values: 20 to 120 MV

MEASUREMENT TECHNIQUES

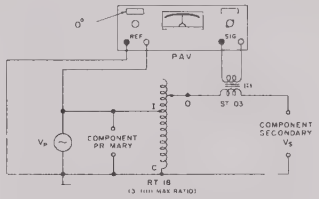

FIGURE 2. *N and* ϕ *Technique # 1.*

PROCEDURE

(1) With PAV at 0°, adjust RT for an inphase null. RT ratio is termed "RT."

(2) Switch PAV to 90°, and record the secondary quadrature voltage (V_{SQ}).

(3) Solve the following equations:

$$\phi = \tan^{-1} \frac{V_{SQ}}{RT\, V_P} \qquad (1)$$

$$N = \frac{RT}{\cos \phi} \qquad (2)$$

Note, that a PDR cannot be used with this technique, unless ϕ is very small or constant. Also, note that RT$>$1 if N$\cdot \cos \phi >$1. If a large number of components of a particular type are to be tested, it may be worth while to make up nomographs relating V_{SQ}, RT, ϕ, and N.

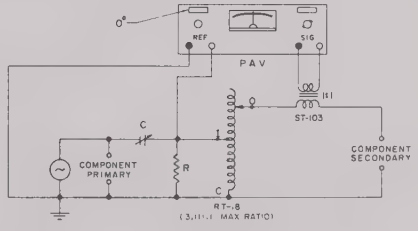

FIGURE 3. *N and* ϕ *Technique # 2.*

PROCEDURE

(1) Adjust RT and C for an inphase and quadrature null with PAV at 0° and 90° respectively.

(2) Solve the following equations:

$$\phi = \tan^{-1} \frac{K}{C} \qquad (1)$$

$$N = RT \cos \phi \qquad (2)$$

where $K \equiv \frac{1}{\omega R}$, and $R \ll Z_{in}$ of RT

Note, that a PDR cannot be used with this technique, unless ϕ is very small or constant. Also, note that RT > 1 if N > cos ϕ. If a large number of components of a particular type are to be tested, it may be worth while to make up nomographs relating C, RT, ϕ, and N.

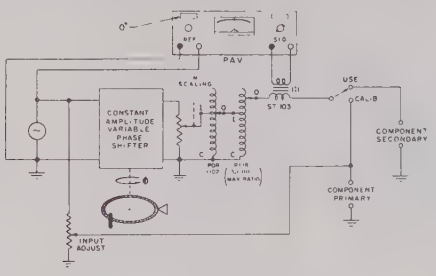

FIGURE 4. *N and ϕ Technique # 3.*

PROCEDURE

(1) Set PDR to 0%, and RT to 1.000. Put "Calib-Use" switch in the "Calib" position.

(2) Adjust "ϕ" dial, and "N Scaling" for an inphase and quadrature null on PAV. Set index on "ϕ" dial to 0°.

(3) Put "Calib-Use" switch in the "Use" position. Adjust "ϕ" dial and RT (or PDR if nominal N is specified) for an inphase and quadrature null on PAV.

(4) N and ϕ may now be read directly on RT (or PDR), and "ϕ" dial.

Note, the constant amplitude variable phase shifter may be of the synchro or resolver type, if care is taken.

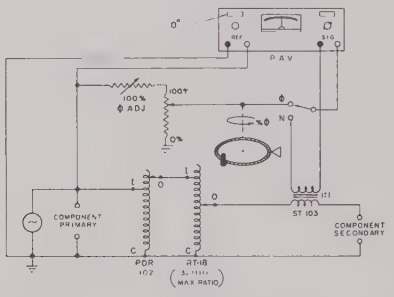

FIGURE 5. *N and ϕ Technique # 4.*

Note, this technique assumes that the secondary quadrature voltage (V_{SQ}) is directly proportional to ϕ, and is, therefore, only valid for small values of ϕ. The "100% ϕ adj" pot is used to set the 100% point on the "% ϕ" pot to correspond, in absolute amplitude, to $V_{SQ\,(max)}$. It is also assumed that cos ϕ = 1.000.

PROCEDURE

(1) Put "ϕ-N" switch in the N position.

(2) Adjust RT (or PDR if nominal N is specified) for an inphase null on the PAV. Switch the PAV to 90°, and note V_{SQ}.

(3) Switch the PAV back to 0°, and place the "ϕ-N" switch in the ϕ position.

(4) Adjust the "% ϕ" pot for a reading on the PAV equal to V_{SQ}.

(5) The RT (or PDR) and the "% ϕ" pot may now be read directly.

Note, if $V_{SQ\,(max)}$ is chosen to equal, say 1°, ϕ may be read directly in degrees from the "% ϕ" dial.

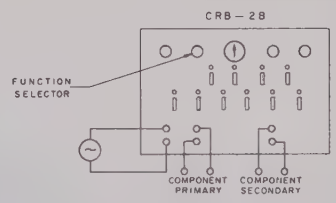

FIGURE 6. *N and ϕ Technique # 5.*

Basically, there are two techniques which may be used with the CRB-2B. The first, and most conventional, technique is to use one of the "rectangular" positions (5°, or 25°) and obtain an inphase and quadrature null. N and ϕ may then be obtained by solving the following equations:

$$\phi = \tan^{-1} \frac{RT_{quad}}{RT_{inphase}} \qquad (1)$$

$$N = \frac{RT_{inphase}}{\cos \phi} \qquad (2)$$

The second technique is to place the "Function" selector in one of the "angle" positions (1° or 10°), and obtain an inphase and quadrature null. ϕ can then be read *directly* from the quadrature RatioTran in terms of degrees. The maximum error in the 10° position is only .05°, and even less in the 1° position. N may be computed from (2) above.

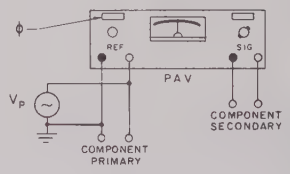

FIGURE 7. *N and ϕ Technique # 6.*

PROCEDURE

(1) Set PAV to 90°, and rotate reference phase shift dial for a quadrature null.

(2) Set PAV to 0° and read $V_{S(max)}$.

(3) ϕ may then be read directly from the reference phase shift dial, and N may be obtained from the following equation:

$$N = \frac{V_{S(max)}}{V_P} \qquad (1)$$

N and ϕ Technique # 7.

For a new improved direct readout technique, see "Measuring Total Transformation Ratio" in Application Note # 12.

Fundamental and Total Null Technique # 1. (See Fig. 7)

PROCEDURE

(1) Read Fund. Null (V_Q) in either the 90° or the Fund. position.

(2) Read Total Null in the Total position.

APPLICATION NOTE NO. 8
TESTING COMPUTATIONAL RESOLVERS

GENERAL

There are two basic classes of resolvers; data-transmission type and computational type. Both types are similar in construction; however, the application and testing of each type is different. The function of data-transmission resolvers is to transmit angular position from one physical location to another. The angular position being transmitted is uniquely and completely defined as a function of the *ratio* of the transmitted voltages to each other. Computational resolvers, however, are generally used in analog computers for the computation of trigonometric equations. Several of these resolvers may be cascaded (along with several amplifiers) in order to solve complex trigonometric equations. Coordinate transformation is a typical application for computational resolvers.

The amplitude of the sine and cosine windings, considered individually at each angle, determines the accuracy of a computational resolver. The "Functional Error" of a computational resolver is a measure of this accuracy, or ability to compute.

The standard equations for computational resolvers are now the same as those for data-transmission type resolvers. Mil-R-23417 specifies the following:

$$E_{S1-S3} = N(E_{R1-R3}\cos\theta - E_{R2-R4}\sin\theta) \quad (1)$$

$$E_{S2-S4} = N(E_{R2-R4}\cos\theta - E_{R1-R3}\sin\theta) \quad (2)$$

for rotor excited units, and

$$E_{R1-R3} = N(E_{S1-S3}\cos\theta - E_{S2-S4}\sin\theta) \quad (3)$$

$$E_{R2-R4} = N(E_{S2-S4}\cos\theta - E_{S1-S3}\sin\theta) \quad (4)$$

for stator excited units.

Within the family of computational resolvers there are two basic types: compensated and non-compensated. Compensated resolvers have internal provisions for minimizing variations in N and ϕ, as a function of temperature and loading.

One form of compensated resolver is the winding compensated type. These units are normally stator input devices, with an extra set of "compensating" windings wound parallel to the two stators. The winding in parallel with S1-S3 is labeled C1-C3 (or S5-S7 by some manufacturers), and the winding in parallel with S2-S4 is labeled C2-C4 (or S6-S8). A common example of this type of computational resolver is the MK 4 variety.

DEFINITIONS

Functional Error is a measure of the accuracy with which the computing function is performed, and is ideally stated as:

$$\frac{E_\Theta}{E_{\Theta=90^\circ}} - \sin\Theta = 0, \quad (5)$$

where E_Θ is the RMS value of the fundamental inphase component of the output voltage at any rotor angle, and $E_{\Theta=90^\circ}$ is the RMS value of the fundamental component of the output voltage at 90°. The actual *difference* in the above equation, multiplied by 100 expresses the Functional Error as a percentage of the output at 90°. The Functional Error at 0° and 90° is zero, by definition.

The output of a .1% computational resolver would be expressed as:

$$E_\Theta = E_{\Theta=90^\circ}\ (\sin\Theta \pm .001). \quad (6)$$

Note, that the allowable error *voltage* at each position is completely independent of Θ. The allowable *angular* error at each position may be computed from the following equation:

$$\Delta\Theta_{(\text{minutes})} \cong 34.4\ \frac{\text{Functional Error (\%)}}{\cos\Theta} \quad (7)$$

Typical values: .2 to .005%

Null Spacing Error is the deviation from 180°, in angular units, of the two inphase null positions of a given winding.

Typical values: minutes

Interaxis Error, or Perpendicularity Error as it's sometimes termed, is the deviation from 90°, in angular units, of the two output windings inphase null positions.

Typical values: minutes

Transformation Ratio and Phase Shift See Note 1. These parameters are usually of particular importance for computational resolver applications. The *equality* of transformation ratios, and the *variation* in ϕ as a function of angle is also of interest sometimes. Winding compensated units, of course, require additional N and ϕ measurements, due to the two additional windings.

Fundamental and Total Null See Note 1.

MEASUREMENT TECHNIQUES

The following techniques illustrate the winding compensated type of computational resolver, since it is the most complex. Although it is not shown, it is understood that the unused stator winding is shorted and the rotor and compensator windings are all terminated with the proper impedances where applicable.

It should be emphasized here, that PDR's (Percent Deviation RatioTran's) *cannot* be used to indicate Functional Error directly in percent!

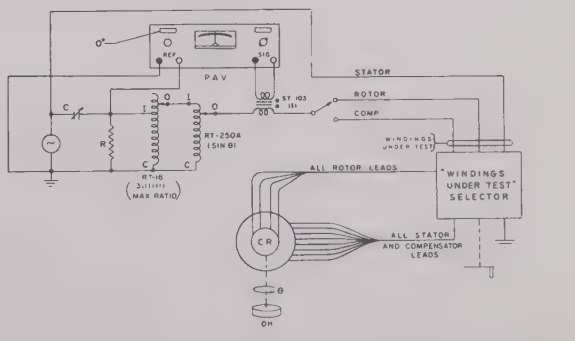

FIGURE 1. *Functional Error, N, and ϕ, Technique # 1.*

Note 1: Refer to Application Note "Measuring Synchro/Resolver Transformation Ratio and Phase Shift, Plus Fundamental and Total Null."

PROCEDURE

(1) Set Sin Θ RT to 90° (unity), and "Rotor-Comp." switch to "Rotor."

(2) Determine N and ϕ from stator to rotor, as explained in Technique #2. (See Note 1)

(3) Set "Rotor-Comp." switch to "Comp."

(4) Determine N and ϕ from stator to comp. as in step (2) above.

(5) Calculate N and ϕ from comp. to rotor from the following:

$$\phi_{C-R} = \phi_{S-R} - \phi_{S-C} \quad (1)$$

$$N_{C-R} = \frac{N_{S-R}}{N_{S-C}} \quad (2)$$

(6) Set C to the value corresponding to ϕ_{S-R}, Sin Θ RT to 0°, and "Rotor-Comp" switch to "Rotor."

(7) Adjust angle of CR (computational resolver) for an inphase null. This is Electrical Zero.

(8) Set Sin Θ RT to 90°, and rotate DH *exactly* 90° from EZ.

(9) Adjust RT for an inphase null, and leave it set to this ratio.

(10) Rotate DH and Sin Θ RT in 5° increments from 90° to 0°. The Functional Error at each angle can be determined directly from the inphase error voltage on the PAV. For example, a .1% unit with an $E_{\Theta=90°} = 20V$ would be allowed $\pm$ 20 MV error at *each* angle. The variable range control on the PAV may be used to adjust 20 MV for full-scale deflection. Thus, it is possible to read Functional Error directly in percent on the PAV meter.

Note that if Technique # 3 (See Note 1) had been used, the N and ϕ from stator to rotor and from comp. to rotor could have been read directly by "calibrating" against the stator and the comp. respectively.

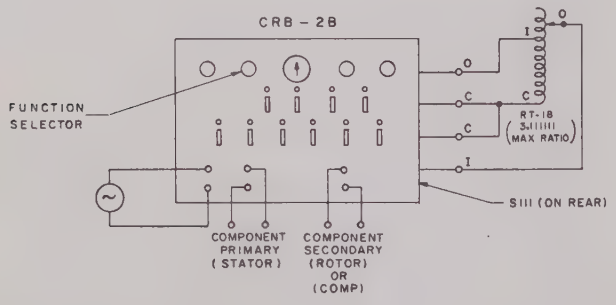

FIGURE 2. *Functional Error, N, and ϕ, Technique # 2.*

PROCEDURE

(1) Determine N and ϕ from stator to rotor, and from stator to comp. as explained in Technique # 5 (See Note 1).

(2) Connect rotor to secondary.

(3) Switch S111 on rear of CRB from "Normal" to "1102/1103", and set "Function" selector to "10° angle" (if ϕ_{S-R} is less than 10°, otherwise use "25° rectangular" position, and obtain quadrature null at each angle).

(4) Set resolver to Electrical Zero. This position can be approximated by adjusting the resolver for an inphase null, if the fundamental null is not too large.

(5) Rotate DH exactly 90° from EZ, set inphase ratio to 1.00000.

(6) Obtain an inphase (with *ext.* RT) and quadrature null. The quadrature RatioTran should now indicate the value of ϕ_{S-R} found in step (1) above. The value of N_{S-R} is now equal to:

$$N_{S-R} = \frac{RT_{EXT}}{\cos \phi} \quad (1)$$

(7) Leave the ext. RT set to this ratio for remainder of test.

(8) Rotate DH in 5° increments from 90° to 0°.

(9) At each 5° increment adjust the inphase RatioTran for a null. The inphase ratio at each angular position should equal the sine of $\Theta \pm$ 1000 PPM for a .1% resolver. The variation in ϕ_{S-R} as a function of angle, can easily be noted on the quadrature RatioTran directly in degrees.

Note, that a Sin Θ RatioTran or a PDR *cannot* be used with this technique.

APPLICATION NOTE NO. 9

TESTING DATA-TRANSMISSION SYNCHROS AND RESOLVERS FOR ANGULAR ACCURACY

GENERAL

The function of data-transmission synchros and resolvers is to transmit angular position from one physical location to another via three or four wires. Synchros have the advantage over resolvers in that one less wire is required; however, it is more difficult to manufacture high accuracy synchros, because of their more complex winding configuration.

DEFINITION — ELECTRICAL ERROR

The angular accuracy of a data-transmission synchro or resolver is briefly defined as the difference between the mechanical position and the electrical position. This difference, expressed in angular units, is referred to as the Electrical Error of a synchro or resolver.

CONTRIBUTING ERRORS

Electrical Error is a result of the combined affect of several measurable errors. The following parameters are all possible contributors:

1. Transformation ratio unbalance.
2. Variation in phase shift.
3. Null spacing error.
4. Inter-axis error.
5. Functional error.

The above parameters are normally not tested individually for data-transmission synchros and resolvers, since it is only the affect that they have on the Electrical Error that is of importance. Note, that is is possible to have a large Functional Error and zero Electrical Error, if the *ratio* of the output voltages to each other is perfect.

By analyzing an Electrical Error curve, a manufacturer can determine the manufacturing errors:

TABLE I

Types of Electrical Error Curves Resulting From Various Manufacturing Errors.

Type of Error Curve	Manufacturing Errors
One cycle	Rotor eccentricity
Two cycle	Stator out-of-round. Unbalanced interwinding capacitance.
High Frequency	Design, since stator and rotor must be made up of laminations having a discrete number of slots.

ELECTRICAL ERROR SPECIFICATION

There are two ways of specifying Electrical Error: (1) *Maximum error* and (2) *Error spread. Maximum error* is the maximum positive or negative Electrical Error, measured over a full 360° of shaft rotation. Depending upon the origin of the error curve, it is possible to obtain two different interpretations of *maximum error.* Military specifications state that the origin of the error curve shall be at EZ, as shown in figure 1 (solid-line curve).

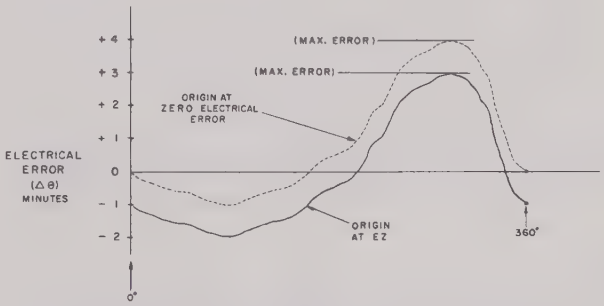

FIGURE 1. *Error Curves With Origin at EZ, and With Origin at Zero Electrical Error.*

Note, that is is possible for a synchro or resolver to have an Electrical Error at EZ. The *maximum error* for the solid-line curve shown is 3 minutes; however, if the origin is chosen at zero Electrical Error, the *maximum error* is equal to 4 minutes. The latter technique is, of course, the simpler and is used whenever possible.

The *error spread* is the sum of the maximum positive and maximum negative errors. Regardless of where the origin is chosen, the *error spread* is the same. Therefore, the origin should always be chosen at zero Electrical Error, since the test procedure is simplified. In the above example, the *error spread* is 5 minutes. As a rule-of-thumb, the *error spread* is approximately equal to 1½ times the *maximum error* from EZ.

ELECTRICAL ZERO

All Electrical Error tests are started at the approximate electrical zero (EZ) point. Coarse EZ of synchros and resolvers is usually indicated by a scribed radius on the exposed shaft end, which is lined up with an arrow stamped on the stator.

This EZ alignment procedure does *not* indicate that the unit will have the proper output phase relationships when the shaft is rotated in either direction. This so-called phasing test is actually performed during the Electrical Error test. If the unit under test and the test instrument must be rotated in opposite directions, the phasing is incorrect.

The following diagrams illustrate the phase relationships at EZ, for the different types of synchros and resolvers:

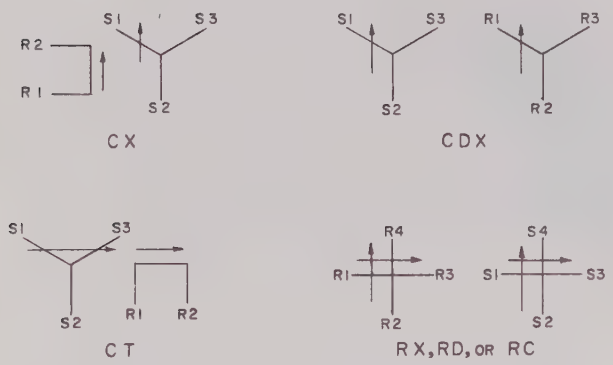

FIGURE 2. *Synchro and Resolver Phase Relationships at EZ.*

TEST CIRCUITS

The procedure for measuring Electrical Error requires either a Synchro/Resolver Standard or a Bridge, as specified in the following table:

TABLE II

Test Instruments Required for Electrical Error Measurement of Various Units.

Type of Unit to be Tested	Test Instrument Required for Electrical Error Measurement			
	SYNCHRO STANDARD	SYNCHRO BRIDGE	RESOLVER STANDARD	RESOLVER BRIDGE
CX		X		
CDX (stator)	X			
CDX (rotor)		X		
CT	X			
RX				X
RD (stator)			X	
RD (rotor)				X
RC			X	

Circuits for the above tests are illustrated in the SS, SB, RS, and RB sections. The R1 and R3 leads shown in the CDX (rotor) and RD (rotor) test circuits may appear to be reversed. This reversal, however, is necessary in order to provide an *increasing* angle to the bridge as the shaft angle is increased. This is a direct result of the fact that differentials *subtract* from the angle applied to their input as their shaft angle is increased (rotated CCW as viewed from the shaft end), if R1 is connected to S1 and R3 is connected to S3 of a control transformer.

TEST METHODS

Two test methods exist for measuring Electrical Error: (1) Proportional Voltage Nulling Method, and (2) Proportional Voltage Gradient Method. Both methods are described in military specifications.

The Proportional Voltage Nulling Method requires the use of the Dividing Head vernier for obtaining an exact null on the Phase Angle Voltmeter. The Electrical Error at each position is then read on the vernier dial.

The Proportional Voltage Gradient Method is several times faster than the nulling method. Since the error voltage gradient of a control transformer, or a control differential transmitter (rotor) is the same at each angle, and is linear for quite a large angular error ($\triangle\Theta = 5°$ causes only .13% error), it follows that the Electrical Error can be read directly in terms of voltage. A Phase Angle Voltmeter with a variable range control is very advantageous for this application, because any *maximum error* may be adjusted to correspond to full-scale deflection. Therefore, it is possible to record Electrical Error at each angle as a percentage of the maximum permissible Electrical Error, or directly from the calibrated scale if the selected full-scale Electrical Error is a multiple of 1 or 3.

The Phase Angle Voltmeter is calibrated by, first, obtaining a null with the Dividing Head vernier, and then purposely introducing (with the vernier) the exact amount of Electrical Error, desired to correspond to full-scale. Next, the variable range control on the PAV is adjusted for full-scale deflection, and the vernier is *then* returned to the null position. The test is performed by advancing the Dividing Head and the Bridge or Standard, in 5° steps, 72 times. At each 5° step the Electrical Error is displayed on the Phase Angle Voltmeter.

It should be noted that a *compensated* Synchro or Resolver Bridge must be employed when using the Proportional Voltage Gradient Method to test control transmitters, or control differential transmitters (rotor).

IMPORTANCE OF ϕ

The reference axis for the phase-sensitive detector must be along the inphase output axis of the synchro or resolver. Otherwise, a portion of the fundamental null voltage (V_Q) will appear to be a part of the inphase output signal. As an example, consider a typical 26V CX with an $E_{S(max)}$ of 11.8V:

$$\text{Fund. Null } (V_Q) = 20\text{MV}$$
$$\text{Phase Shift } (\phi) = 10°$$

If the input axis is chosen as the reference axis, a small amount of signal voltage is required to cancel the portion of V_Q which is "inphase" with the input.

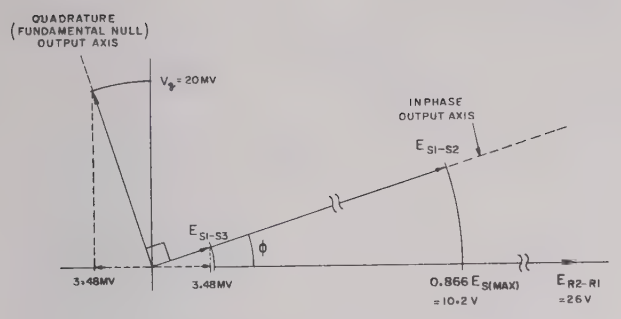

FIGURE 3. *CX Outputs Near EZ.*

As can be seen from figure 3, the amount of signal voltage (E_{S1-S3}) necessary is:

$$E_{S1-S3} = \frac{20 \times \sin 10^\circ}{\cos 10^\circ} = 3.54 \text{ MV}. \qquad (1)$$

This corresponds to a value of Θ equal to:

$$\Theta = \sin^{-1} \frac{3.54\text{MV}}{11.8\text{V}} \cong 1' \qquad (2)$$

Therefore, the angle of the CX is approximately one minute from actual EZ.

A rule-of-thumb frequently used for specifying fundamental null voltage is that it shall be less than 50% of the maximum permissible Electrical Error in terms of voltage. The error voltage gradient of the CX in the above example is:

$$\frac{\triangle V}{\triangle \Theta} = 11.8 \sin 1^\circ = 206 \text{ MV/degree} \qquad (3)$$

Therefore, the approximate permissible *maximum error* of the CX in the example above is:

$$\triangle\Theta = \frac{2 \times 20 \text{ MV}}{206 \text{ MV/degree}} \cong .2^\circ = 12' \qquad (4)$$

Total null is, as a rule-of-thumb, allowed to be 50% greater than the fundamental null. This would amount to 30MV in the above example.

$\frac{\triangle V}{\triangle \Theta}$ MAGNITUDE AND PHASE

The magnitude of the error voltage gradient $\frac{\triangle V}{\triangle \Theta}$, or the "sensitivity" can be calculated from the equations in the following table for various error-sensing devices.

TABLE III

Error Voltage Gradient Equations for Various Error-Sensing Devices.

Error-Sensing Device (Stator Input / Rotor Output)	Error Voltage Gradient $\left(\frac{\triangle V}{\triangle \Theta}\right)$, or "sensitivity"		
	Volts/Degree	MV/Minute	μV/Second
CDX (stator),*	$.0151NE_{S\,(max)}$	$.252NE_{S\,(max)}$	$4.20NE_{S\,(max)}$
CT*	$.0174E_{R\,(max)}$	$.291E_{R\,(max)}$	$4.85E_{R\,(max)}$
SB,**	$.0174NE_{S\,(max)}$	$.291NE_{S\,(max)}$	$4.85NE_{S\,(max)}$
RB**	$.0174E_{R\,(max)}$	$.291E_{R\,(max)}$	$4.85E_{R\,(max)}$
RD (stator), RC	$.0174NE_{S\,(max)}$	$.291NE_{S\,(max)}$	$4.85NE_{S\,(max)}$
	$.0174E_{R\,(max)}$	$.291E_{R\,(max)}$	$4.85E_{R\,(max)}$

* The transformation ratio (N) of a CDX or CT, as defined by Mil-S-20708B, is equivalent to the rotor voltage at maximum coupling divided by 0.866 $E_{S(max)}$.

** Gertsch Compensated Synchro and Resolver Bridges have a transformation ratio (N) of 1.000. Note, that an SB with an N of 1.000 is equivalent to a CT with an N of 1.155.

By observing the phase of the error voltage ($\triangle V$) relative to the reference voltage, it can be determined whether the equivalent angular position of the input signal to the error-sensing device (Θ_{elect}) is greater than, or less than, the angular position of the error-sensing device (Θ_{mech}).

TABLE IV

Phase Relationship Between Error Voltage and Reference Voltage for Various Error-Sensing Devices.

Error-Sensing Device	Θ_{elect} greater than Θ_{mech}
CT, SB	$\triangle V_{R2-R1}$ out-of-phase with E_{R2-R1}
CDX (stator)	$\triangle V_{R3-R1}$ out-of-phase with E_{R2-R1}
RD (stator), RB, RC	$\triangle V_{R3-R1}$ inphase with E_{R2-R4}, or $\triangle V_{R4-R2}$ inphase with E_{R3-R1}

This phasing information is extremely important when trying to differentiate between true-null and false-null which are 180° apart. For example, if an SB is connected to the output of a CX, and the angle of the SB (Θ_{mech}) is purposely *increased* beyond null, Θ_{elect} is less than Θ_{mech} and $\triangle V_{R2-R1}$ should be *inphase* with E_{R2-R1}. If it is out-of-phase, the angle displayed on the SB is 180° from the true angular position of the CX.

APPLICATION NOTE NO. 10
AN IMPORTANT MEMO TO ALL SYNCHRO AND RESOLVER TEST EQUIPMENT USERS

Purpose of this memo

The following paragraphs will show that Gertsch Ratio Tran Synchro and Resolver Bridges actually cause *less* loading error than the 10K resistive-type bridges, when both types are compared under actual-use conditions. The several other advantages of Ratio Tran Bridges, such as immunity to environmental conditions, extremely high ratio accuracy, low output impedance, etc., will not be discussed.

What is meant by loading errors?

Loading errors result whenever the angular position being transmitted is altered by the application of a load, such as a bridge.

What are the basic causes of loading errors?

Basically, there are two causes of loading errors: (1) load impedance unbalance, and (2) source impedance (Z_{SS}) unbalance. The load impedance unbalance for our purpose here is represented by the bridge, and is normally stated in per cent. Resistive-type (10K) bridges are normally 0.01%, and inductive-type bridges are typically 10%. The source impedance unbalance normally ranges from 0.2% to 2%, depending on the component.

How are these unbalances related to the loading errors?

The error due to either type of unbalance is *inversely* proportional to the load impedance, and *directly* proportional to the source impedance. In terms of an equation then, for synchros:

$$\Delta\Theta \text{ max} \cong \frac{4\, Z_{SS}\,(\alpha_1 + \alpha_2)}{3\, Z_l},$$

where –

$\Delta\Theta$ max ≅ Peak (+ or -) loading error in seconds-of-arc.

Z_{SS} = Stator impedance of CX with rotor shorted, in ohms.

α_1 = Z_{SS} unbalance of CX, in percent.

α_2 = Bridge input impedance unbalance in percent.

Z_l = leg impedance, in 1000's of ohms.

NOTE, that the 4/3 term disappears for resolvers. The following chart gives several examples:

Type of synchro bridge	Z_{SS} = 75 ohms (50 ohms in series with each leg)		Z_{SS} = 750 ohms (500 ohms in series with each leg)		Z_{SS} = 7500 ohms (5000 ohms in series with each leg)	
	α_1 = 0.2%	α_1 = 2%	α_1 = 0.2%	α_1 = 2%	α_1 = 0.2%	α_1 = 2%
500K, 10% inductive-type	2 sec.	2.4 sec.	20 sec.	24 sec.	200 sec.	240 sec.
10K, 0.01% resistive-type	2 sec.	20 sec.	20 sec.	200 sec.	200 sec.	2000 sec.

How is the Z_{SS} unbalance of a component specified?

The Z_{SS} unbalance of a component is normally specified in terms of "Z-seconds". The Z-second error of a component is measured by applying a load equal to ten-times the source impedance (Z_{SS}), and measuring the resultant loading error in seconds. Therefore, it is possible to convert the Z-second error of a component directly to a percentage unbalance. From the above equation it can be shown that a component with a 100 Z-second error has an output impedance unbalance of 1%, which is quite typical.

Are large values of Z_{SS} common?

Values above 1000 ohms are quite uncommon, in fact the majority of components in use today are probably less than 100 ohms.

APPLICATION NOTE NO. 11 *

ANGLE TO DIGITAL CONVERTERS FOR SYNCHRO AND RESOLVER TESTING

Introduction

Data-transmission synchros and resolvers are rotary transformers used to transmit angular position information over a distance.

Synchros contain three identical windings spaced 120° from each other. Connected in a wye configuration, their outputs are carried over three wires. A resolver has two separate windings, spaced 90° apart, with four output wires. Either device converts angular shaft position into electrical voltages, whose relative amplitudes uniquely define an electrical angle analogous to the mechanical shaft angle.

The electrical schematics of a synchro and a resolver are shown in Figure 1. The output voltages of a synchro transmitter (CX), Figure 1a and Figure 1b, are:

$$E_{S1\text{-}S3} = E_{S(max)} \sin \theta_e$$
$$E_{S3\text{-}S2} = E_{S(max)} \sin (\theta_e + 120°)$$
$$E_{S2\text{-}S1} = E_{S(max)} \sin (\theta_e + 240°)$$

Where θe = electrical angle

The output voltages of a resolver transmitter (RX), Figure 1c are:

$$E_{S2\text{-}S4} = E_{S(max)} \cos \theta_e$$
$$E_{S1\text{-}S3} = E_{S(max)} \sin \theta_e$$

Errors and Measurement Considerations

The most important characteristic of a data transmitter synchro or resolver is its angular accuracy. The angular error is the difference between the actual mechanical shaft angle (θs) and the electrical angle θe calculated from the ratios of the output voltages. Electrical errors can be caused by unbalance of the transformation ratios of the windings, variation in phase shift, and the presence of harmonics, among other factors. These errors must be determined by measuring the electrical angle in any of several ways, and comparing it to the mechanical shaft angle.

An instrument for measuring the electrical error of a synchro or resolver should have the following characteristics:

- It should display the electrical angle automatically in digital form, to an accuracy as great as 0.001°. A binary coded output (BCD) of its reading should be available for interface with a printer or external computer.

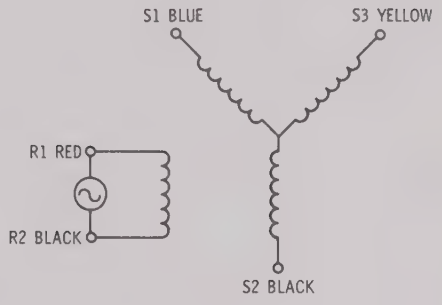

Figure 1a — Synchro Transmitter (CX) Outputs

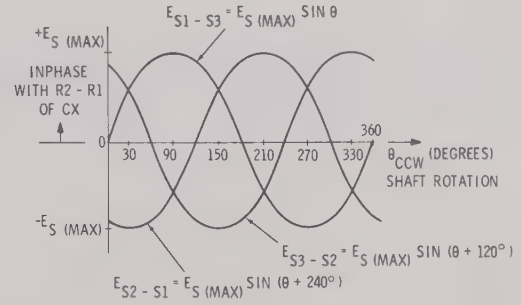

Figure 1b — Synchro Transmitter Phase Relationships

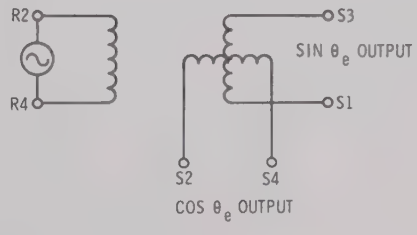

Figure 1c — Resolver Transmitter (RX) Outputs

*First edition 8/69. Second edition 5/72

- It should operate over a wide range of input voltages (e.g., $E_{S(max)}$ between 8 volts and 128 volts).
- It should be possible to pre-set a desired angle, manually or by means of an external digital control signal, with the instrument then producing a voltage proportional to the difference between the pre-set electrical angle and the actual electrical angle.
- The instrument should be able to measure electrical angle for both a synchro and a resolver.
- The input to the instrument should not load the test circuit sufficiently to produce a significant error in its reading.
- Since a basic test typically involves 72 angular measurements at 5° intervals, the instrument should operate as rapidly as possible.

The Singer Instrumentation ADC family of analog-to-digital converters was developed at the Gertsch facility to satisfy the above requirements.

Description of the ADC Family of Instruments

The Singer "ADCs" or Angle to Digital converters consist of a main frame and one or more plug-in modules, and meet all of the stated requirements for accurate measurement of the electrical angle of a synchro or resolver transmitter output.

The model ADC-1, with the Model CC-1 Converter Control unit installed, displays the electrical angle of a synchro or resolver automatically, with an accuracy of 0.001° and a conversion time of 250 milliseconds. The front panel readout consists of seven NIXIE® tubes displaying the angle in degrees (decimal notation) to a resolution of 0.0005°. Simultaneously a BCD output is available on the rear apron for external print-out.

With the Model CC-7 Converter Control Unit installed, either the Model ADC-1 or ADC-2 becomes, in effect, a very precise control transformer (CT) which may be programmed manually or by external electrical means to any desired angle.

All basic functions are programmable, including switch-over from synchro to resolver input, input voltage level, and the built-in 45° self-test feature. There is also a "HOLD" command which holds a reading indefinitely until released.

The Model ADC-2 (Figure 2), with the Model CC-2 Converter Control plug-in, is identical to the Model ADC-1/CC-1, but has a five digit NIXIE® tube display with both accuracy and resolution of 0.01°, and a conversion time of only 200 milliseconds.

Description of the Resolver Bridge

In the ADC instruments the output voltages of a synchro or resolver are applied to a high resolution resolver bridge. The basic bridge configuration is shown in Figure 3. The resolver bridge consists of two transformers, one with sine taps, and one with cosine taps. The sine voltage is applied across one transformer and the cosine voltage is applied across the second. The transformer with the sine voltage input is tapped to provide an output voltage proportional to the cosine. The second ratio transformer is tapped to provide an output voltage proportional to the sine.

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Figure 2 — The Model ADC-2 Angle/Digital Converter

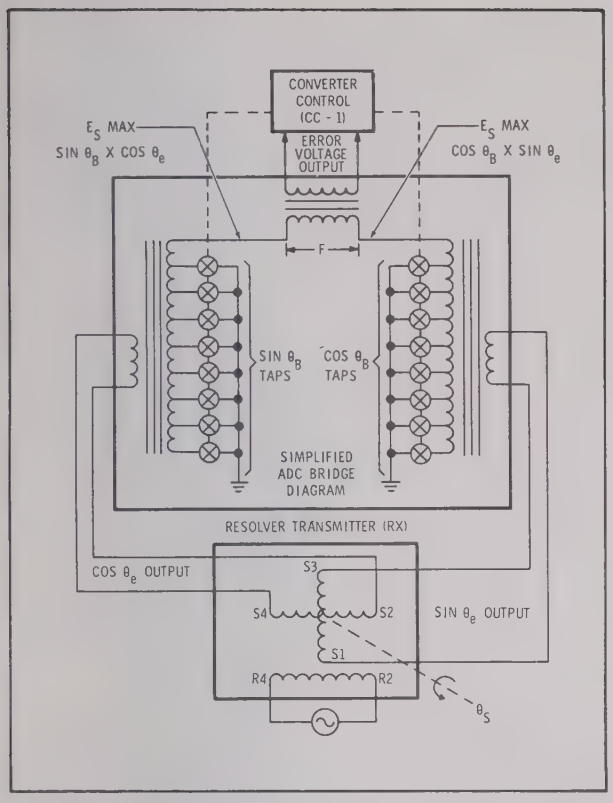


Figure 3 — Resolver Bridge Voltage Comparator Circuit

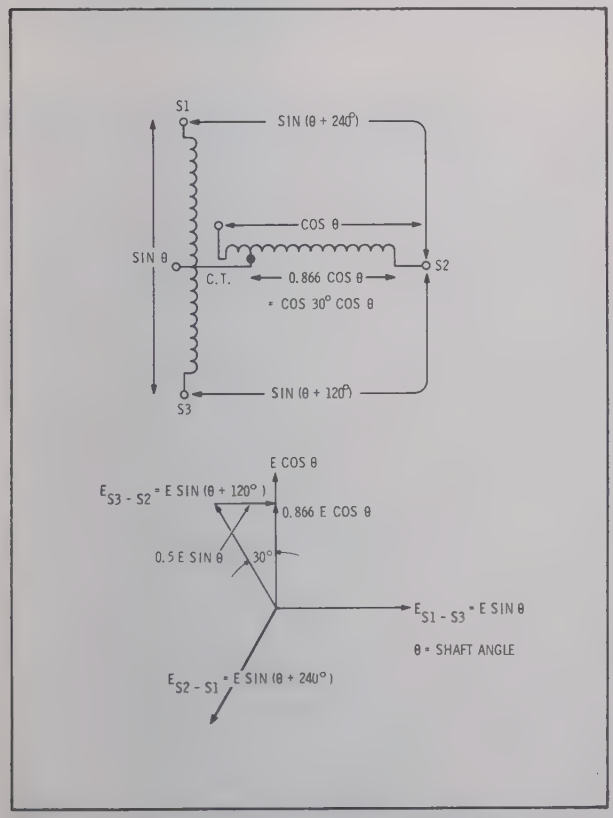


Figure 4 — Scott-Tee Transformation & Spatial Vector Relationships

GLOSSARY OF TERMS

Synchro Control Transmitter (CX)
In a CX, the system reference voltage is applied across a single rotor (primary) winding. Three wye-connected stator (secondary) windings are spatially displaced 120 degrees from each other. Stator induced voltages are all electrically in-phase (or 180 degrees out of phase) with the input voltage, and vary in magnitude according to the rotor position.

Synchro
A CT is similar in configuration to a CX, with the primary and secondary functions reversed. When the single rotor (secondary) winding is oriented along the zero axis, its output is a null. Changes in rotor angular position generate a voltage which is used to drive a servo system.

Resolver Control Transmitter (RX)
A resolver has two stator and two rotor windings. One rotor, serving as the primary, is excited by the reference voltage. The other rotor winding is short-circuited. The two stator windings deliver output voltages proportional to the sine and cosine of the shaft angle θ.

θ is the mechanical shaft angle, measured positively for ccw rotation from electrical zero (EZ) as viewed from the shaft end. Ideally, θ is equal to the electrical angle computed from the output voltages from the synchro or resolver windings.

$\emptyset$ is the electrical (time) phase shift between the reference voltage and the output voltages from the synchro or resolver windings. It is measured at the first position of maximum coupling in the positive direction (ccw) from electrical zero (EZ).

For any angle, both voltage outputs of the bridge are proportional to a sine and a cosine product, obtained from the resolver outputs. Whenever $\theta_e = \theta_B$, the resultant output is a null. Whenever a bridge and the RX are set to different angles, an error voltage output results,

where: $E_{out} = E_{S(max)} [\sin \theta_B \cos \theta_e - \sin \theta_e \cos \theta_B]$
$= E_{S(max)} \sin (\theta_B - \theta_e)$

The error voltage output of this system is supplied to a converter control unit, which adjusts the transformer ratios to minimize the error voltage. Through a Scott-Tee transformer arrangement (Figure 4) this same bridge circuit can be used to measure the angular position of a synchro transmitter (CX). The vector diagram of Figure 4b shows the conversion from three 120° spatially separated components to two 90° spatially separated components in time phase. The electrical position of a synchro can be determined from amplitude ratios in a three-wire output as shown in Figure 1. At zero degrees, the secondary voltage across S1 and S3 is zero and the secondary voltage across S1 and S2 is close to maximum. The ratio (A) between the two voltages $A = \frac{S1 - S3}{S1 - S2}$ is therefore zero. At 60 degrees, the two secondary voltages are equal; therefore, the ratio is unity.

The voltage across S1 and S3 is a sin θ voltage and this voltage is used as the sin θ voltage of a resolver. To derive the cos θ voltage, two transformers are interconnected into a Scott-Tee as shown in Figure 4a. The voltage from the center tap of the sin θ transformer to S2 is equal to cos 30° × Cos θ. This voltage is inserted at a cos 30° tap of the second transformer. The voltage developed across the full transformer is therefore equal to cos θ.

Bridge Circuit

Figure 5 shows a block diagram of the bridge portion of the ADC. In the resolver mode, the sine and cosine inputs from an RX are applied directly to transformers T1 and T2. These transformers are tapped to adjust the bridge to the maximum amplitude level of the incoming voltage. In the synchro mode, these same two transformers form the Scott-Tee shown in Figure 4a as previously described.

The full bridge consists of transformers T3 through T11 plus the resistive ladder networks. To achieve the required resolution of θ_R, the actual bridge circuit is more complicated than shown in Figure3 . The block diagram in Figure 5 shows the major circuits required to achieve 0.0005° resolution. A simplified derivation of the trigonometric voltages is shown in Figure 5. All of the constants associated with transformation ratio have been ignored. At the 0.01° level and below, the tangent function is essentially linear and therefore BCD resistive ladder networks may be used in place of transformers without significant degradation of accuracy. Transformer taps are selected by transistor switches which change ground points rather than switching outputs directly. The voltage ratios of the incoming signal repeat for every quadrant, however, the ratios exist across a different combination of input terminals. The 90° and 180° bits arrange the decade transformers for proper interpretation of the signal.

Nulling Process

The error voltage output is applied to an over/under null comparator through a gated sample and hold circuit. The sample and hold gate is controlled by the synthesized reference which also functions as the clock source for the logic circuitry. The successive approximation method is controlled by the over/under null comparator.

When the bridge circuit passes through a null point, the error voltage output passes through a 180° phase reversal. The comparator uses this phase reversal as the indication of a null point. The process of nulling is accomplished by making successively smaller comparisons. If a given comparison passes null, the step is negated and the process continues to the next less significant bit.

To see how this is accomplished, assume an actual input of 37°. Assume also that the correct quadrant has been determined. The process will continue as follows:

(1) 80° tested ——over null——negate 80°
(2) 40° tested ——over null——negate 40°
(3) 20° tested ——under null——accept 20°
(4) 20° +10° tested ——under null——accept 10°
(5) 30° +8° tested ——over null——negate 8°
(6) 30° +4° tested ——under null——accept 4°
(7) 34° +2° tested ——under null——accept 2°
(8) 36° +1° tested ——under null——accept 1°

Successive tests will test for smaller increments. The last "decade" is really only one bit, i.e., 0.0005°. With this bit, the two quadrature bits and five decades of four bits each, the total is 23 bits.

Error Sources

Input Impedance

As with any high-accuracy instrument of this type, high input impedance is a major design goal. This allows the bridge to be connected in parallel with an actual load or to be connected into a working system without additional

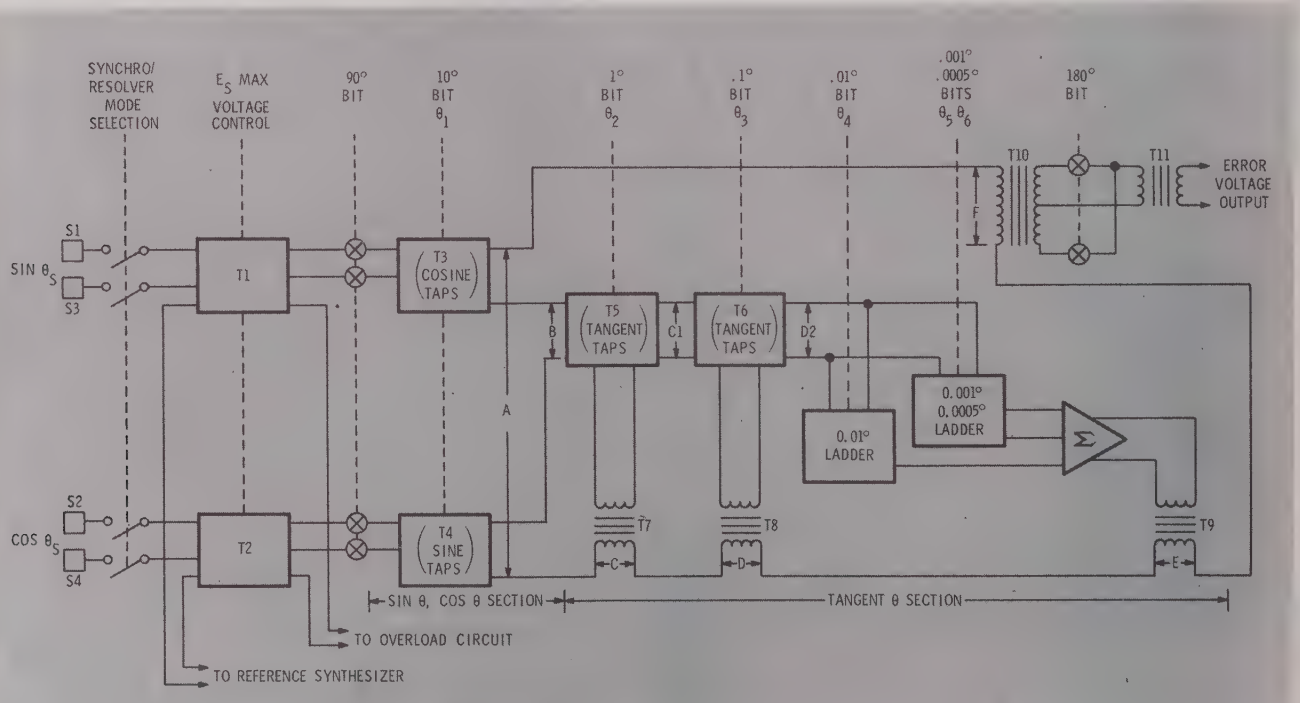


Figure 5 — Bridge Circuit Block Diagram

loading. However, for synchros and resolvers with significant values of output impedance, input impedance unbalance further complicates the design problem.

Since the bridge compares two voltages, any difference in load impedance changes the ratio of these voltages, and introduces a nulling point error. The equivalent angular error due to input impedance unbalance, may be calculated from the following approximation:

$$\theta_{MAX}\,(\text{sec}) = \text{Real Part of } \frac{[Z_{SS}\,(\text{ohms})]\,[Z'\text{ unbalance }(\%)]}{Z'\,(\text{k ohms})}$$

Where Z′ is the average value of $Z_{S3\text{-}S1}$ and $Z_{S2\text{-}S4}$, the bridge input impedances, and Z′ unbalance is equal to the difference between Z_{S3-S1} and Z_{S2-S4} divided by Z′. Z_{SS} is equal to the stator (S1—S3, or S2—S4) output impedance of the resolver under test, with the rotor shorted.

A Figure-of-Merit may therefore be defined for a resolver bridge, relating input impedance and input impedance unbalance:

$$\gamma \equiv \frac{Z\,(\text{k ohms})}{Z\text{ unbalance }(\%)}$$

Thus,

$$\theta_{MAX}\,(\text{sec}) = \text{Real Part Of } \frac{Z_{SS}\,(\text{ohms})}{\gamma}$$

Two isolation transformers in the input step down the ratio, and thus reduce the reflected admittance from the secondary to primary. Consequently, it is possible to obtain higher values of input impedance. Furthermore, these isolation transformers eliminate the need for common connections between the sine and cosine inputs.

An isolation transformer is also provided for the bridge output to minimize problems from ground loops.

$A = \text{Sin}\,(\theta_s - \theta_1)$

$B = \text{Cos}\,(\theta_s - \theta_1)$

$C = B \times \text{Tan}\,(\theta_2)$

$C_1 \cong B \times \text{Tan}\,(1°)$

$D \cong B \times \text{Tan}\,(\theta_3)$

$D_2 \cong B \times \text{Tan}\,(0.1°)$

$E \cong B \times \text{Tan}\,(\theta_4 + \theta_5 + \theta_6)$

$F = A + C + D + E$

$= \text{Sin}\,(\theta_s - \theta_1) - \text{Cos}\,(\theta_s - \theta_1) \times \text{Tan}\,(\theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6)$

$$= \frac{\text{Sin}\,[\theta_s - (\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6)]}{\text{Cos}\,(\theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6)}$$

$$F \cong \frac{\text{Sin}\,[\theta_s - \theta_b]}{\text{Cos}\,(\theta_t)}$$

θ_b = Bridge Angle $= \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6$

θ_t = Tangent Transformer Angle $= \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 < 10°$

When $\theta_b = \theta_s$ Then F = 0 (Null)

Proper Reference Phase

Conventional synchros and resolvers, as transformers, have a time leading phase shift (Ø) from primary to secondary. Phase shift is measured at the position of maximum coupling and typical values range from 0° to 20°.

According to MIL-Specs[1,2,3], the reference axis for the phase-sensitive detector must be along the in-phase output axis of the synchro or resolver; otherwise, a portion of the fundamental null voltage (V_Q) will appear to be a part of the in-phase output signal. As an example, consider a typical 26V CX with $E_{S(max)}$ of 11.8V:

Fundamental Null (V_Q) = 20mV

Phase Shift (Ø) = 10°

If the input axis is chosen as the reference axis, a small amount of signal voltage is required to cancel the portion of V_Q which is "in-phase" with the input.

As can be seen from Figure 6, the amount of signal voltage ($V_{S_1-S_3}$) necessary is:

$$V_{S1-S3} = \frac{20 \text{ x } \sin 10°}{\cos 10°} = 3.54 \text{ mV.}$$

This corresponds to a value of θ equal to:

$$\text{Ø} = \arcsin \frac{3.54\text{mV}}{11.8} = 1'$$

In this example, the electrical position of the CX is apparently displaced by one minute-of-arc. By using a reference voltage with the same phase-shift angle, the true angular position is measured. A unique feature of the Singer ADC instruments is the synthesized reference voltage which is generated within the system for the phase-sensitive detector. This detector reference voltage automatically adjusts to the phase shift through the device or system under test and eliminates the error caused by the combination of quadrature and phase shift. This method of deriving the reference voltage also differentiates between true null and false null which are located 180° apart mechanically. The synthesized reference is generated from three inputs: a portion of the sine, a portion of the cosine, and a system reference signal. Both sine and cosine inputs are required to ensure an amplitude of sufficient magnitude. The system reference voltage is used to determine the proper quadrant for the synthesized reference.

Noise, Harmonics, and Magnetic Fields

Synchros and resolvers are basically variable coupling transformers, but they are not necessarily ideal transformers. Their output can include noise, harmonics, and quadrature (synonomous with fundamental null)[4], all of which can obscure or even change the null point. For this reason, several features have been included in the ADC so that 0.001° accuracy can be realized under actual operating conditions.

As the bridge approaches null, the total error voltage decreases and therefore noise and harmonics become a more

1. MIL-S-20708 2. MIL-R-21530 3. MIL-R-23417

4. See Application Note # 7.

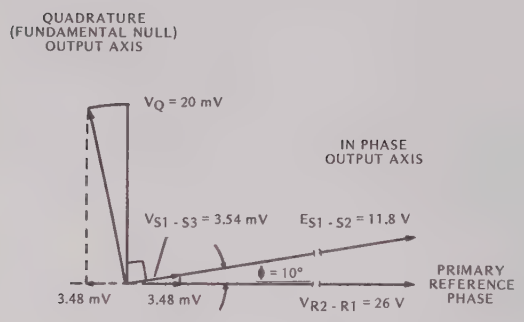


Figure 6 — Null Voltage Relationships

critical factor. The higher the resolution, the more important the effect of noise and harmonics. Close to null, their instantaneous effect can be an apparent phase reversal. Passive filters in both error voltage and reference voltage channels minimize this effect and allow greater resolution and accuracy. Another very important consideration for 0.001° accuracy is the problem caused by external magnetic fields. If this problem is ignored, large errors can result. For example, if an unprotected bridge is mounted near an instrument with a large power transformer, the power line frequency will be coupled into the bridge and appear as a false signal. In the ADC, careful shielding and other design precautions have been taken to minimize this problem.

Ratio Accuracy and Stability

Accuracy within the bridge[5] circuit depends on the ratio accuracy of the transformer taps. The equation usually found for the voltage ratio is SIN θ_S COS θ_B = COS θ_S SIN θ_B. Therefore, the SINE/COSINE tap approach is a straightforward solution to identifying the ideal transformer taps. The task has been to solve the complete problem of transformer windings with 0.0005° resolution. Computer-aided design of hand-wound transformers represent the Gertsch method.

Trigonometric tapped transformers could be used as a model problem for computer-aided design. Total number of turns, transformation ratio, tap ratios, core material, and input impedance are some of the variables which must be combined for an optimum solution. A computer program was used for these calculations.

Once an optimum solution was reached, the hand-winding techniques developed for the RatioTran ensure the desired accuracy. Ratio accuracies of 1 part per million have been verified by the National Bureau of Standards. Since the transformers are passive devices, they are rugged, reliable, and relatively immune to environmental changes.

5. See Application Note # 6.

Operating Characteristics

At the heart of the unit is the toroidal transformer, a passive device which is stable with time. All silicon, solid state switching is used. The control logic makes maximum use of IC's in a modular, plug-in board concept.

A 45° self-test circuit allows the unit to be subjected to a rigorous complete test of function at the touch of a switch. From the system reference, an isolation transformer derives the ideal voltages of a resolver at a 45° position. The system should null to this signal and display a 45° angle ±0.001°.

As a further self-test feature, an OPERATING indicator is incorporated in the system. This indicator lights when all externally supplied inputs are proper and the nulling circuit is active. Synchronized by the clock source, the control logic directs a bit-by-bit serial comparison for detecting the over/under null condition. The shift register output is encoded into an 8-4-2-1 BCD, parallel code by means of a gate array. Set/reset control of the storage register flip-flops providing a decade-by-decade approach to null detection. The storage register output controls transformer taps through a set of solid state switches. At the conclusion of the 23 bit sensing program, an update display pulse is presented to the display register. This allows the previous bridge null setting to be displayed while the succeeding nulling cycle is in process. The storage register, therefore, eliminates the digit blur normally associated with successive approximation techniques. At the end of each conversion cycle a print pulse is generated. A HOLD control allows the bridge to be stopped at any selected angle. A START control allows the operator to start the conversion cycle at any time desired.

Figure 7 is a block diagram of the ADC-1/CC-1 showing the control logic, reference synthesizer, and phase sensitive detector (comparator). The ADC may be used as an independent test piece with front panel control of functions and a local display, or interfaced with digital equipment for automated systems. To simplify interfacing the BCD output is DTL, compatible with the standard 0 volt "0" and 5 volt "1" logic levels. Current sink input logic is also used.

Summary of Features

The Singer ADC family of Angle-to-Digital Converters offer an unparalleled combination of accuracy and resolution (two to four times better than competitive instruments) and automatic operation. They are fully compatible with digital computers and printers for automated test systems. Among their advanced design features are a synthesized reference signal whose phase automatically compensates for phase shift in the devices under test, harmonic filters to eliminate errors due to distortion components. Overload protection against excessive input voltage, and an exceptionally high input impedance to reduce loading errors.

The concept of separating the instrument into a basic bridge unit and a plug-in control module assures the purchaser of a Singer ADC-1 or ADC-2 against obsolescence in the face of changing measurement requirements. The presently available Converter Control Modules, CC-1, CC-2 and CC-7, cover a large variety of applications; additional plug-ins which may be developed in the future for

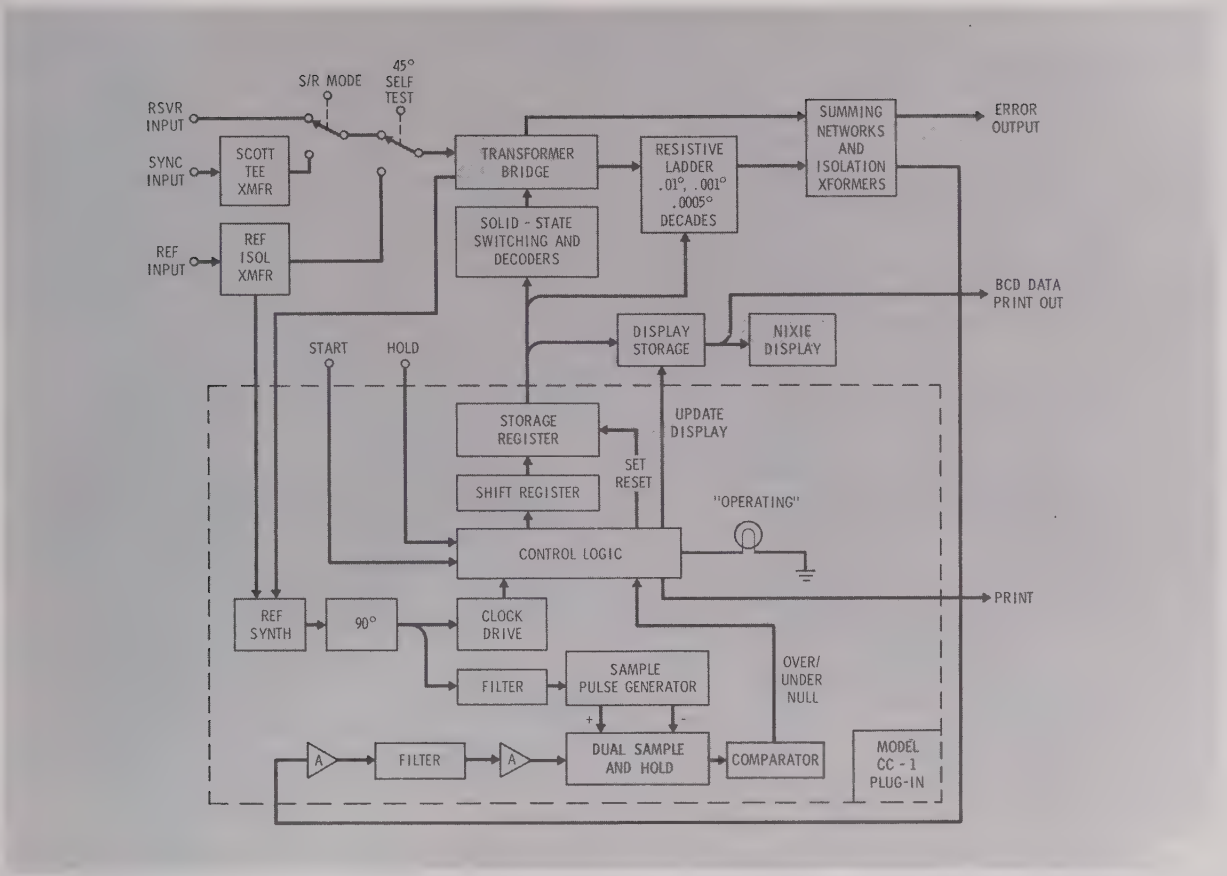


Figure 7 — Model ADC-1 Simplified Block Diagram

other measurement needs will be usable without obsoleting the basic ADC unit.

Applications[6]

With the Model CC-1 converter control module installed, the Model ADC-1 is ideally suited to rapid testing of data-transmission synchros and resolvers. Using a dividing head to set the shaft position in 5° increments, the 250 millisecond conversion time of the Model ADC-1 makes available highly accurate electrical angle data in little more time than is required for the operator to adjust the shaft angle. The automatic nulling circuit of the Model ADC-1 and its digital angle readout minimize the chance of operator error.

The compatibility of the Model ADC-1/CC-1 with a digital computer makes it possible to completely automate the testing of synchros and resolvers. This versatile system is equally adaptable to laboratory use, receiving inspection, and production line testing.

Because of its high input impedance, it can be used for accurate, automatic or manual testing of systems, including servos, radar antenna mounts, inertial platforms, gyroscopes, and aircraft indicators. System errors, drift, and stability can be measured and recorded through the fully programmable BCD controlled inputs and outputs of the Model ADC-1/CC-1. By connecting a Model PAV-4 Phase Angle Voltmeter to the error voltage terminals of the Model ADC-1/CC-1, readings can be made of in-phase and quadrature null voltages at each test setting.

With the Model CC-7 converter control module installed, the Model ADC-1 becomes a very precise programmable control transformer (CT). Any desired electrical angle may be programmed into it, either electrically or manually, and external servos used to drive a synchro control transmitter (CX) to the same angle, as indicated by a null voltage at the Model ADC-1 error voltage terminals. Its 0.001° accuracy and 0.0005° resolution make it compatible with the requirements of critical data-transmission systems.

For less exacting measurement requirements, the Model ADC-2/CC-2 offers the same operating flexibility, with 0.01° accuracy and resolution, at a substantial cost saving.

6. See Application Note # 9.

Specifications

ADC-1, 2

Accuracy	ADC-1 ADC-2	0.001° 0.01°
Resolution	ADC-1 ADC-2	0.0005° 0.01°
Signal Input	Voltage ($E_{s(max)}$). Frequency Synchro Input Impedance Resolver Input Impedance	8 - 128 Vrms 400 Hz ± 5% 750k ohms (min) 500k ohms (min)
Reference Input	Voltage Input Impedance	8 - 128 Vrms 300k ohms (min)
Error Voltage Output	ADC-1 ADC-2	35 - 70 mV/° 0.547 $E_{s(max)}$ mV/°
Input Programming Logic, on Rear Connector	"1" "0"	+5V, +1V, −1V (0.01 mA load) 0V, +0.45V, −0V (7 mA sink out)
Data Output Logic (DTL), on Rear Connector	"1" "0" Coding	+5V, +1V, −2.4V (0.04 mA source). 0V,+0.45V, −0V (8 mA sink) BCD - 8, 4, 2, 1 all decades

CC-1, 2

Conversion Time	CC-2: CC-1:	200 ms (max) 250 ms (max).
Hold Time (front or rear)		0 to infinity
Print Pulse (on rear)		1 ms at end of display update.
Harmonic Suppression		40dB (min)
Reference Phase Range		−3° to +100°

ADC-1, 2/CC-1, 2

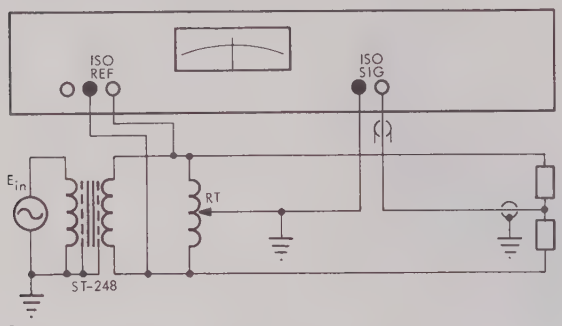
Temperature Range (operating)	0 to 50°C
Size	5¼" H x 19" W x 18" D
Weight	Approx 40 lbs
Power	115 - 230V ± 10%, 50 - 420 Hz, Approx. 100 W

APPLICATION NOTE NO. 12

A POTPOURRI OF UNIQUE PAV-4 APPLICATIONS

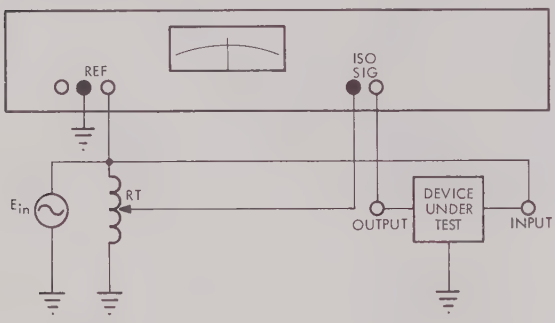
12–1
High Impedance Ratio Measurements

To measure the attenuation or transformation ratio of a device with high output impedance, use the configuration below.



12–2
Measuring Very Small Phase Angles

To measure phase angles smaller than the resolution capability of the PAV-4, use the circuit shown below.



PROCEDURE:

1) Adjust RT for an in-phase null
2) Measure quadrature voltage V_{SQ} at null
3) Compute ϕ:

$$\phi = TAN^{-1} \frac{V_{SQ}}{RT \times E_{in}}$$

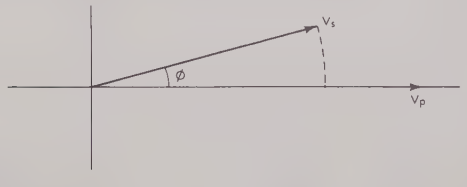
For small angles:

$$\phi_{RADIANS} = \frac{V_{SQ}}{RT \times E_{in}}$$

$$\phi_{DEGREES} = \frac{V_{SQ}}{57.3 \times RT \times E_{in}}$$

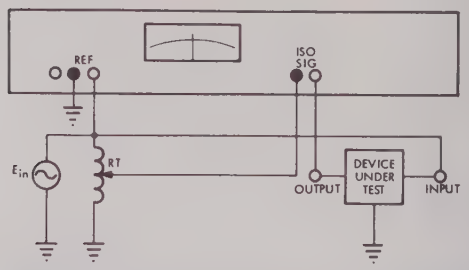
12–3
Measuring TOTAL Transformation Ratio

The Total Transformation ratio (TTR) of a device is defined as the ratio of the maximum secondary voltage (V_s) to the primary or excitation voltage (V_p).



$$TTR = \left|\frac{V_s}{V_p}\right|$$

To achieve maximum accuracy and sensitivity, a Ratio Tran is used in conjunction with a PAV in a bridge circuit.



Normally, the reference axis for the measurement is chosen as 0°. When this is done, the TTR must be computed from the following equation:

$$TTR = \frac{RT \text{ (at null)}}{COS\phi}$$

Where ϕ is the phase shift from primary to secondary. This additional computation can be avoided normally by simply setting the reference axis equal to $\phi/2$, instead of 0°. Therefore, to make direct reading TTR measurements:

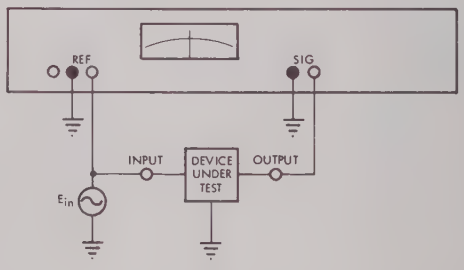
1) Measure ϕ with PAV.
2) Set PAV reference axis to $\phi/2$.
3) Adjust RT for null.
4) TTR = RT setting at null.

12–4
Measuring Propagation Time Delay

The propagation time delay (sometimes referred to as envelope delay or group delay) of a device is proportional to its phase slope *versus* frequency characteristic:

$$t_D = \frac{1}{360} \times \left(\frac{\Theta_2 - \Theta_1}{f_2 - f_1}\right) \text{ seconds,}$$

where Θ is in degrees, and f is in Hertz.



Example: The phase change of a 400Hz bandpass filter is 10° over a differential frequency range of 10Hz (405 - 395) —

$$t_D = \frac{1}{360} \left(\frac{10°}{10Hz}\right) = 2.78 \text{ ms}$$

The time delay is 2.78 ms at 400 Hz.

12–5
Measuring Phase Angles >360°

If the phase angle to be measured is greater than 360°, the PAV will read $\Theta_1 = (\Theta_T - 360N)°$. To measure the total phase angle Θ_T at frequency f_1, note the phase angle reading at f_1, then increase the frequency slightly to f_2. Note Θ_2.

$$\Theta_T = \Theta_1 + 360N = \left(\frac{\Theta_2 - \Theta_1}{f_2 - f_1}\right) f_1 \text{ degrees.}$$

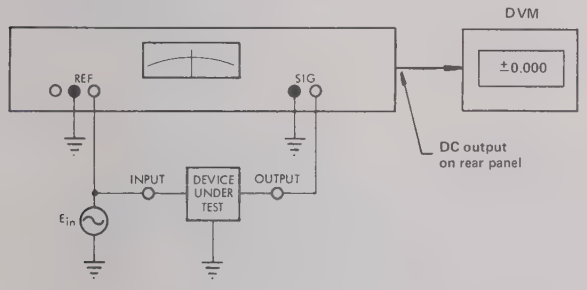
Note: this equation is only valid for linear phase devices.

12–6
DIGITAL READOUT FOR HIGHER ACCURACY

Since the PAV-4 has a single-ended (one side grounded) DC output, a conventional 0 – 1 VDC integrating-type digital voltmeter may be used to improve the accuracy or ease of readout. Integrating-type voltmeters are generally recommended, because of their long-term averaging characteristics.

Normally, the front panel analog meter on the PAV-4 contributes approximately one-half of the rated 2% accuracy specification. As a rule-of-thumb, range-to-range errors are 0.25%, phase-sensitive mode linearity is 0.2% of full-scale, and "standard" voltmeter (fundamental and total) mode linearity is 1% of full-scale. Therefore, phase-sensitive accuracies of 0.2% maybe expected on a single range calibrated at full-scale.

Because of this high degree of phase-sensitive linearity accuracy, phase-shift measurements with accuracies of 0.1° are possible. This is accomplished by simply measuring the inphase and quadrature components, and then computing their ratio. The phase angle is then equal to the arc-tan (or arc-cot) of the ratio. The necessity for this computation can be avoided by adjusting the larger of the two readings for 1.000. This may be accomplished by either adjusting the excitation to the device under test, or by using the variable attenuator control concentric with the PAV-4 VRMS switch.



12–7
PRECISION FREQUENCY COMPARATOR

The frequency of two oscillators may be compared with a PAV-4. Frequency differences from as small as 1 part in 10^{12} or less, to as large as 1 Hz can be measured over a nominal frequency range of 30 Hz to 0.3 MHz with appropriate plug-ins. Very small frequency differences (less than 0.0001 Hz can be determined in one minute, because of the PAV-4's high phase sensitivity (F.S. = 5.7°) and variable phase reference. The following equation describes the relationship between phase change, observation time internal, nominal frequency, and the frequency error in Hertz or proportional parts.

$$\frac{\Delta f}{f} = \frac{\Delta \emptyset}{360° f \Delta t}$$

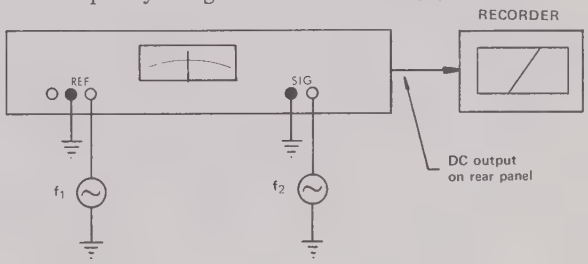
where,

f = Nominal frequency in Hertz.
Δf = Frequency difference in Hertz.
$\Delta \emptyset$ = Phase change in degrees.
Δt = Time, in seconds, required for $\Delta \emptyset$.

For example, two 100 KHz oscillators can be determined to be within 1 part-per-billion (PPB) in less than 1 minute. A recorder may be used on the output of the PAV-4 to indicate short-term frequency difference.

In order to make a frequency comparison and/or determine whether the signal frequency is higher or lower than the reference frequency:

1. Adjust the PAV-4 reference axis for a positive peak indication on the meter.
2. **Advance** the quadrant switch 90°, and adjust the reference axis for a precise null.
3. Observe whether the meter moves + (to the right) or – with time. If the meter moves +, then the signal frequency is higher than the reference.



4. Offset the reference axis by some convenient amount, say 10°. Note, if the initial drift direction was +, then advance the reference phase (10°) so as to cause an initial – reading on the meter.
5. Note the amount of time that elapses between step 2 above (initial null setting) and the instant the meter crosses through zero again.
6. Solve the preceding equation for Δf.

12–8
FREQUENCY–SELECTIVE VOLTMETER

When combined with a VF plug-in the PAV-4 makes a very narrowband frequency-selective voltmeter, which may be used over a wide frequency range. The VF-1 provides continuous frequency coverage from 300 Hz to 10 KHz, while the VF-2 covers the range from 10 KHz to 0.3 MHz. The SF and MF plug-ins, however, are limited to single frequency operation (±5%) between the frequency limits of 30 Hz and 10 KHz.

As can be seen from Figures 1 and 2, the bandwidth in the phase-sensitive modes is extremely narrow. Also, in the fundamental mode, the VF plug-ins have a much narrower bandwidth than the SF or MF plug-ins.

It should be noted, however, that the VF plug-ins have no image-rejection. Furthermore, for optimum spurious response, the reference source should be free of spurious signals when using the VF plug-ins.

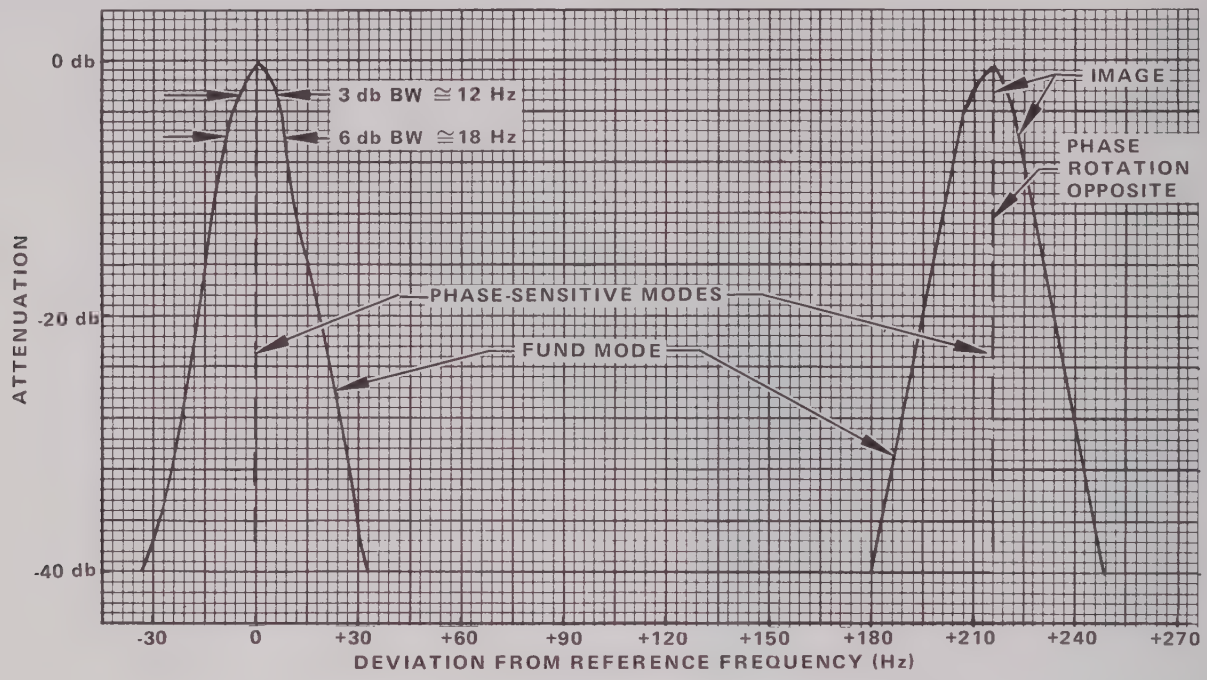


FIGURE 1. VF-1 FREQUENCY RESPONSE (TYPICAL)

NOTES

1. Multiply above frequencies by 33.3 to obtain VF-2 frequency response.
2. 60 dB BW = 100 Hz, and 6 dB BW = 18 Hz. Therefore, the shape factor = 5.5.
3. Image response is always 2 x IF frequency above the tuned frequency.

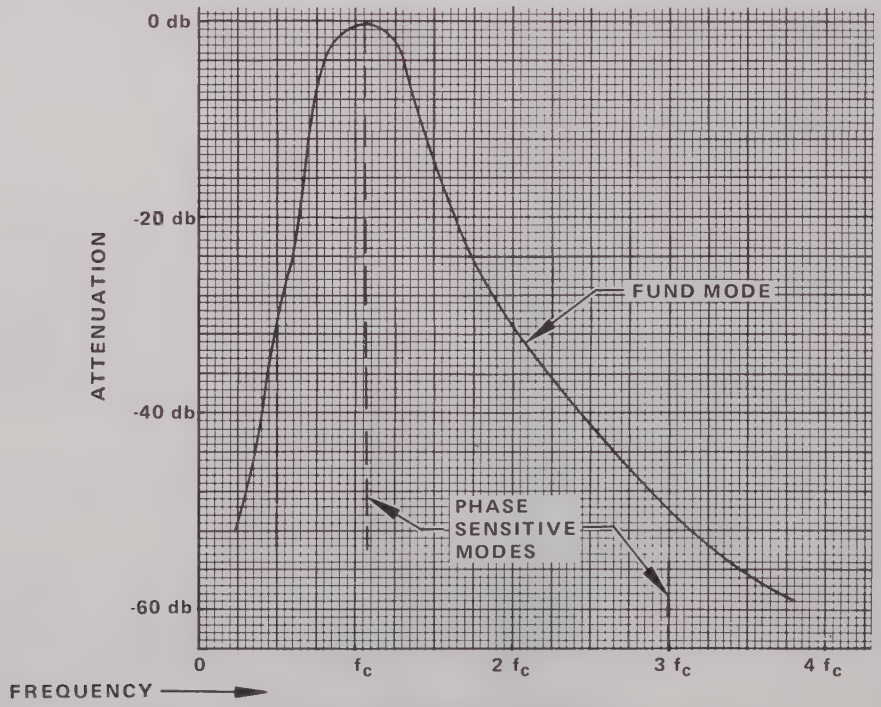
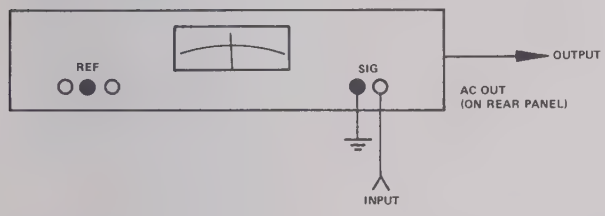


FIGURE 2. SF-1, MF-1 FREQUENCY RESPONSE (TYPICAL)

12-9

TUNED AMPLIFIER (-65 to + 55 dB)

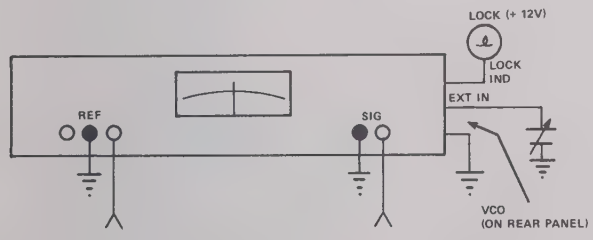
When combined with an SF of MF plug-in, the PAV-4 in the Fundamental Mode can be used as a wide range tuned (30 Hz to 20 KHz) amplifier. The gain (or attenuation) is determined by the setting of the VRMS switch. On the 300 uV scale, it is possible to amplify a signal 55 dB (200 mv rms maximum output). As the VRMS is switched to higher ranges, the gain decreases 10 dB per step.



12–10

REMOTE FREQUENCY PROGRAMMING

The frequency of the VF plug-ins may be remotely programmed from the rear panel of the PAV-4 with an adjustable DC voltage. Nominally, -5.6V corresponds to f(min) and +1.4V equals f(max). As with manual control from the front panel, the desired frequency should always be approached from the high side. In other words, the tuning voltage should be varied from + to -, until a lock indication (+12V) is obtained.



12–11

TRUE POWER MEASUREMENTS (MICROWATTS to WATTS)

When the current and the voltage waveforms in a circuit are sinusoidal, the true power is equal to:

$$P = V_{in}\ 1 \cos \Theta$$

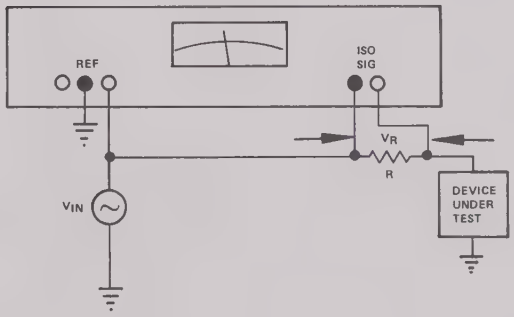
where Θ is the phase difference between the voltage and the current.

The PAV-4 in the inphase (0°) mode with the aid of a 1 ohm $\pm$ 1% resistor can be used to measure 1 cos Θ directly, since:

$$1 \cos \Theta = \frac{VR \cos \Theta}{R} = V_R \cos \Theta$$

Therefore, a $V_R \cos \Theta$ reading of 15 mv would correspond to 15 ma.

Note, an isolated PAV-4 is not required if either the generator or the device under test can be floated off of ground, thus allowing R to be put in the low side.



APPLICATION NOTE NO. 13

DIRECT READOUT RESOLVER PHASE SHIFTER

The unique design configuration of Gertsch Resolver Bridges (5° or Decade Models) allows them to be used as a constant amplitude direct readout phase-shifter when driven from a two-phase generator*.

When connected as shown in Figure 1, the output voltage across R1-R3 is equal to the absolute value of the two input voltages shifted in phase (relative to the input across S1-S3) by an amount equal to the angle set in on the bridge. The output voltage $E\angle\Theta$ or $V_{R3\text{-}R1}$ is derived in the following equations:

$$
\begin{aligned}
V_{R3\text{-}R1} &= V_{S4\text{-}S2} \sin\Theta + V_{S3\text{-}S1} \cos\Theta \\
&= E \sin(\omega t + 90°) \sin\Theta + E \sin\omega t \cos\Theta \\
&= E \cos\omega t \sin\Theta + E \sin\omega t \cos\Theta \\
&= E \sin(\omega t + \Theta)
\end{aligned}
$$

It will be noted that the above derivation is based on two assumptions about the two-phase generator:

1. $|E\angle 0°| = |E\angle 90°|$. If this is not true, an equivalent angular error will result which will peak at 45°, 135°, 225°, and 315°. A 1% inequality causes a 0.3° error.
2. $V_{S4\text{-}S2}$ leads $V_{S3\text{-}S1}$ by 90°. Any orthogonality error causes an angular error to peak at 90°, and 270°. The magnitude of the maximum angular error will be directly equal to the orthogonality error.

The output impedance of a typical Gertsch Resolver Bridge is specified as 400 + j 40; therefore, only high impedance loads should be driven directly from the output of the bridge. Loading errors will cause a constant offset error, which may be calculated, if the impedances are known.

*Weston Model CVO-100, or equivalent.

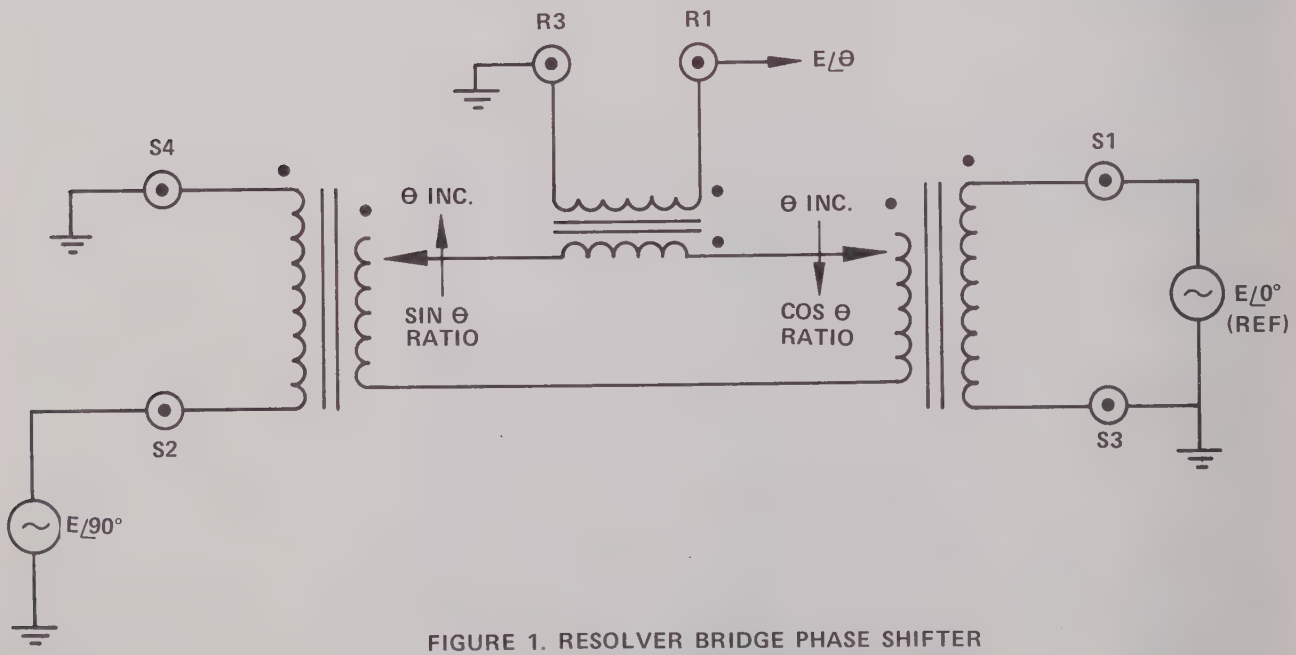


FIGURE 1. RESOLVER BRIDGE PHASE SHIFTER WITH DIRECT READOUT OF ANGLE

APPLICATION NOTE NO. 14
MEASURING INPHASE RATIOS WITH HIGH-RESOLUTION RESOLVER BRIDGES

Frequently it is desired to measure the ratio of two AC voltages which are nearly inphase or 180° out-of-phase with each other. The conventional approach to this problem is to use a Complex Ratio Bridge (CRB), or a Ratio Tran and a null-detector. It is not generally realized, however, that a high resolution (Decade) Resolver Bridge may also be used. To understand how this is possible, it is necessary to review some basic resolver theory.

A resolver may be thought of as a variable coupling transformer which generates two voltages proportional to the sine and cosine of its shaft angle. Therefore, the angle transmitted by a resolver is uniquely defined by the ratio of the two outputs to each other. Since one is the sine and the other is the cosine, this ratio is equal to the tangent of the angle.

Resolver bridges are devices which display the angle being tranmitted by a resolver directly in degrees. Therefore, the ratio of the two input voltages may be computed by taking the arc tan of the angle being displayed at null. Figure 1 illustrates how two voltages whose ratio is to be measured should be connected.

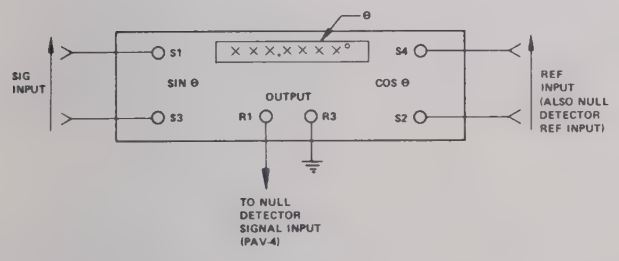


FIGURE 1. INPHASE RATIO BRIDGE USING A MANUAL DECADE RESOLVER BRIDGE

At null, $V_{sig}/V_{ref} = \text{Tan}^{-1}\ \theta$. The new HP-35 Pocket Calculator is ideal for the purpose of taking the arc tan of the angle displayed at null. Because of the 4-terminal input configuration of Gertsch Decade Resolver Bridges, it should be noted that the signal and reference inputs need not have a common ground. Furthermore, because of the built-in quadrant switching, it is not necessary to reverse leads, etc when measuring out-of-phase voltage ratios. Inphase ratios will be indicated by an angular readout in the 1st quadrant, while out-of-phase ratios will be in the 4th quadrant.

If semi-automatic ratio measurements are desired, an automatic bridge such as the Gertsch ADC-1 may be used. However, it will still be necessary to manually compute the arc-tan, unless a tan look-up ROM is used on the BCD outputs. See Figure 2. Note, that the ADC reference may be taken from the signal input or $V_{S3\text{-}S1}$, if $V_{S2\text{-}S4}$ is less than the minimum reference input required by the ADC-1 (8 VRMS). The arc cot must, however, be taken instead of the arc tan.

The accuracy of a resolver bridge is normally specified in angular units. The maximum equivalent inphase ratio error of a bridge is 10PPM per second. Therefore, a 2 second bridge has a maximum ratio error of 20PPM or 0.002%.

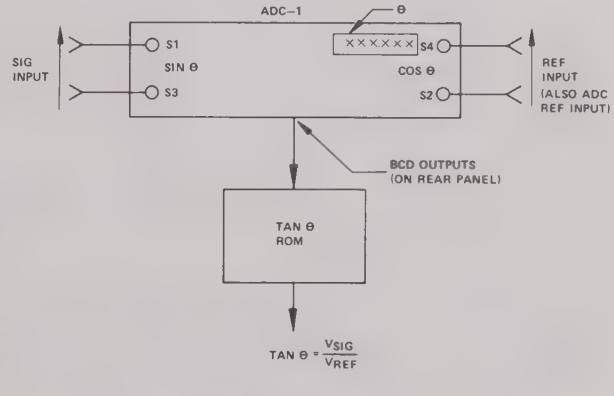


FIGURE 2. AUTOMATIC INPHASE RATIO BRIDGE USING AN ADC–1

APPLICATION NOTE NO. 15

TRANSFORMER VOLTAGE DIVIDER RATIO ACCURACY AT HIGH AUDIO FREQUENCIES

SUMMARY

The accuracy of transformer voltage dividers is susceptible to precise theoretical analysis. The best example is provided by T. L. Zepf et al in the September issue of IEEE Transactions, 1963 on Instruments and Measurements. This report presents a generalized approach to the most accurate type of transformer voltage divider. Formulas relating both inphase and quadrature ratio accuracy frequency and "figure of merit" are developed from a mathematical model. The figure of merit is easily determined from input and output impedance measurements. Validity of mathematical model is supported by test data.

VOLTAGE DIVIDER MODEL

The model used to develop accuracy formulas is shown in Figure 1. A four section divider is shown for illustrative purposes only. This model most nearly represents a divider manufactured by winding (m) strands multifilar on a single toroid core.

FORMULAS

The resulting formulas fall into 3 groups as follows:

1. Relationship of the interstrand capacity (Cw) to the reflected capacity (Cwr) measured at the input terminals.

2. Relationship of the short circuit impedance (2xZs) to the short circuit tap impedance [Zsc(n)] measured at the tap (n).

3. The relationship of the voltage ratio error [Δ VR(n)] measured at the tap (n) as a function of frequency.

EXPERIMENTAL PROCEDURE

A single decade divider was constructed at Singer – Gertsch using normal manufacturing procedures as described under "Voltage Divider Model". Before connecting the strands to form a divider, two types of measurements were made:

1. Each interstrand capacitance was measured. There were 45 such values in a 10 strand divider. The measurement was made with a 3 terminal capacitance bridge by guarding 8 strands and measuring the capacitance between the 2 remaining strands. The results are shown in Figure 3. The average value was 191.1pf.

2. Each strand short circuit impedance was measured. There are 10 values. The measurement was made with an impedance bridge by shorting 9 strands and measuring the impedance of the 10th strand. The results are not plotted because all values were essentially identical.
Zs = 0.3033 + j 0.0066 at 400 Hz.

The windings were then connected as a divider and three groups of measurements were made to compare against calculated values obtained from previous tests.

1. The input impedance was measured as a function of frequency and the reflected capacity calculated from values above the resonant point. The results are plotted in Figure 4. Almost perfect correlation was obtained.

2. The short circuit tap impedance was measured and compared against the calculated values. This was done at 400 Hz. The results are plotted in Figure 5. Again, almost perfect correlation was obtained.

3. Finally the tap ratio error was measured at 10 KHz using a Singer – Gertsch Model RT – 23 divider as a comparison standard. The results are plotted in Figure 6. The quadrature correlation is excellent but the inphase error was some 50% less than calculated. An explanation for this result was found in the inaccuracies associated with measuring the reactive component of the strand short circuit impedance.

CONCLUSIONS

The tests indicate that the formulas are accurate within the accuracy associated with the measurements. An important result is that the formulas can be used to predict ratio accuracy at high frequencies. It is only necessary to make a simple input impedance measurement and simple short circuit test at a convenient frequency. The value (Zs/Zw) calculated from these measurements could also be used as a "figure of merit" for dividers of this type.

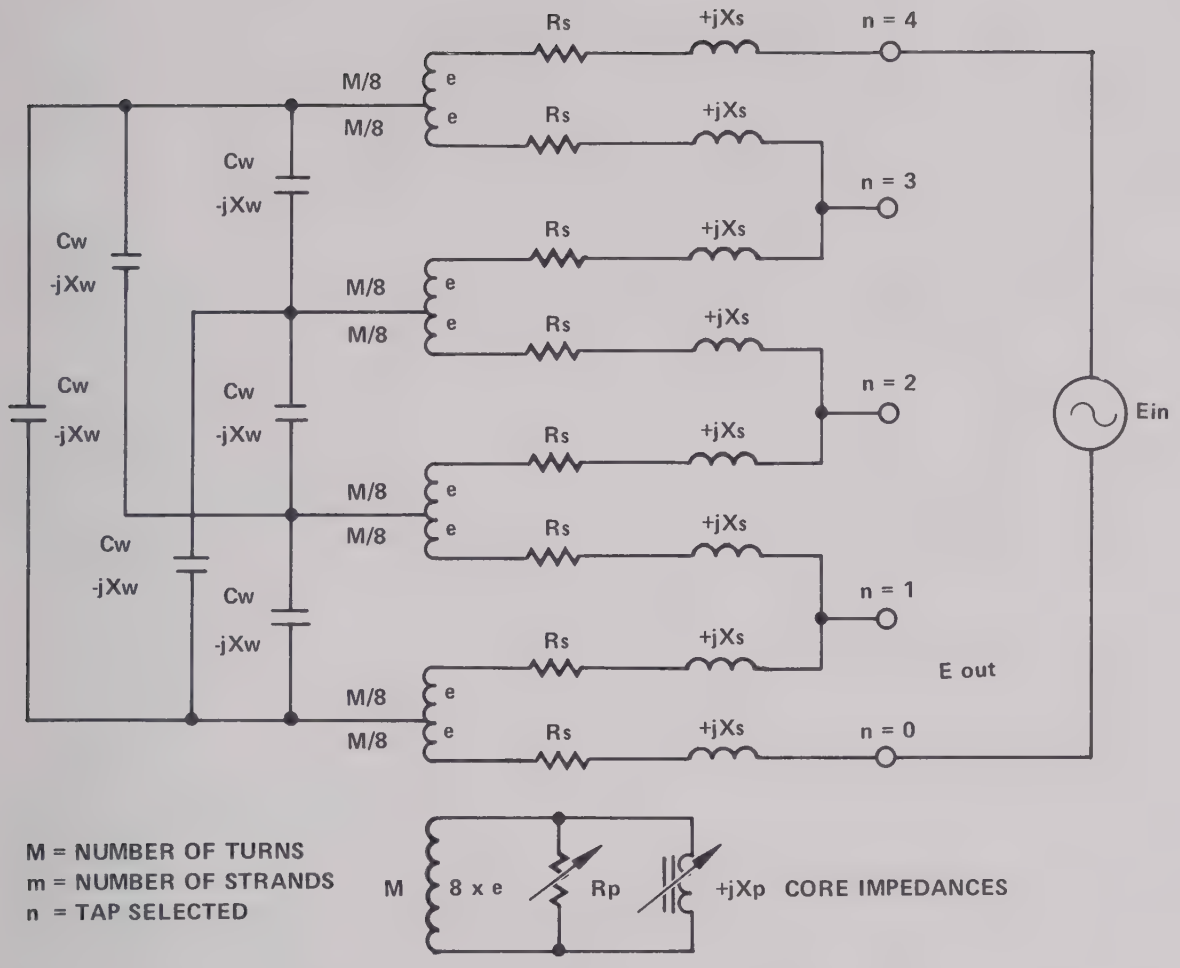


FIGURE 1. TRANSFORMER VOLTAGE DIVIDER

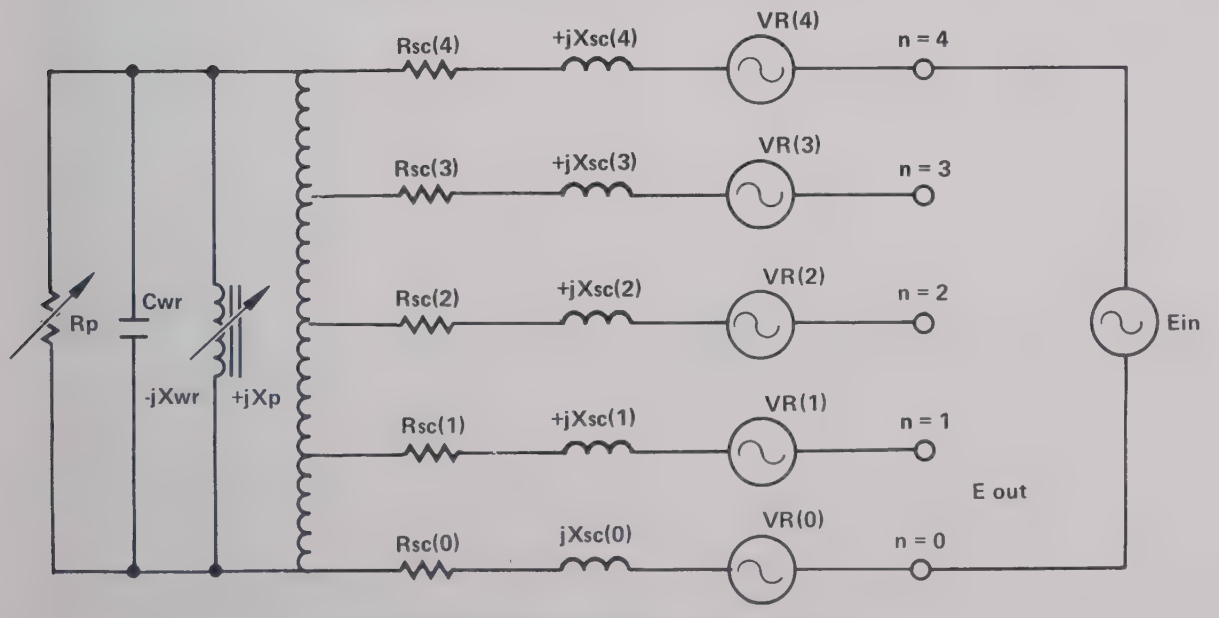


FIGURE 2. EQUIVALENT SCHEMATIC DIAGRAM

FORMULAS

THE FORMULAS WERE OBTAINED BY ASSUMING $Z_s \ll X_w$

FIRST GROUP: Interstrand Capacity (Cw)
Reflected Interstrand Cap (Cwr)

(1-1) $Cwr = Cw \times (m^2 - 1)/12$

(1-2) $Xw = Xwr \times (m^2 - 1)/12, X = 1/(2 \times pi \times f \times C)$

(1-3) $(m^2 - 1)/12 = 82.5, m = 10$

(1-4) Total number of $Cw = \sum_{i=1}^{i=m-1} i$

SECOND GROUP: Short Circuit Strand Impedance (Z_S)
Short Circuit Tap Impedance [Zsc(n)]

(2-1) $Zs = Rs + jXs$

(2-2) $Zsc(n) = Zs \times K(n)$

(2-3) $K(n) = 2\,(n/m)(m-n)$

THIRD GROUP: Voltage Ratio Error [ΔVR(n)]

(3-1) $VR(n) = E\,out/E\,in = n/m + \Delta VR(n)$

(3-2) $\Delta VR(n) = (Zs/Zw) \times \Sigma(n)$

(3-3) $\Sigma(n) = (n/6)(m^2 - 3mn + 2n^2)$

(3-4) $\Sigma(s) = (m^3/6)(S - 3S^2 + 2S^3)$

(3-5) $S = (n/m)$ = (Dial or Tap Setting)

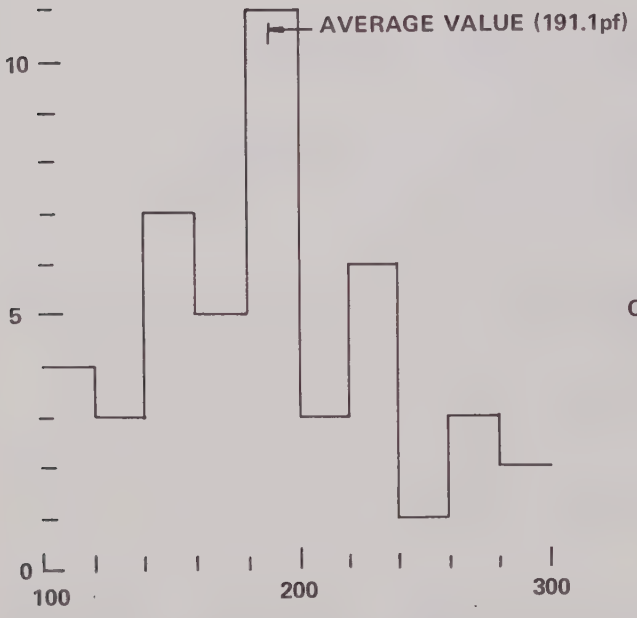
(3-6) $\Delta VR_{(n)} = (-A \times f^2 \times j\,B \times F)\Sigma_{(n)}$

A = Xs/Xw at Freq. f_0, B = Rs/Xw at Freq. f_0

f_1 = Test Frequency, f = f1/f0 (Normalized Frequency)

n =	0	1	2	3	4	5	6	7	8	9	10
K(n) =	0	1.8	3.2	4.2	4.8	5.0	4.8	4.2	3.2	1.8	0

n =	0	1	2	3	4	5	6	7	8	9	10
X(n) =	0	12	16	14	8	0	-8	-14	-16	-12	0



HORIZONTAL: CAPACITANCE (PICO FARADS)
VERTICAL: NUMBER OF VALUES PER 20pf INCREMENT
FIGURE 3. DISTRIBUTION OF INTERSTRAND CAPACITY

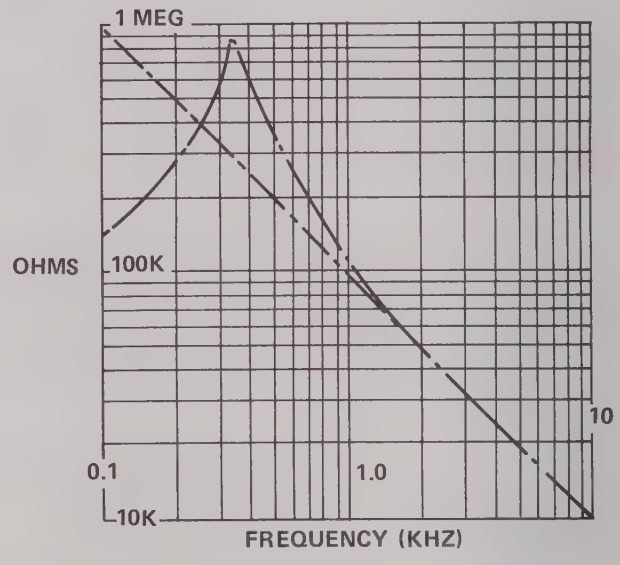


FIGURE 4. INPUT Z vs. FREQUENCY

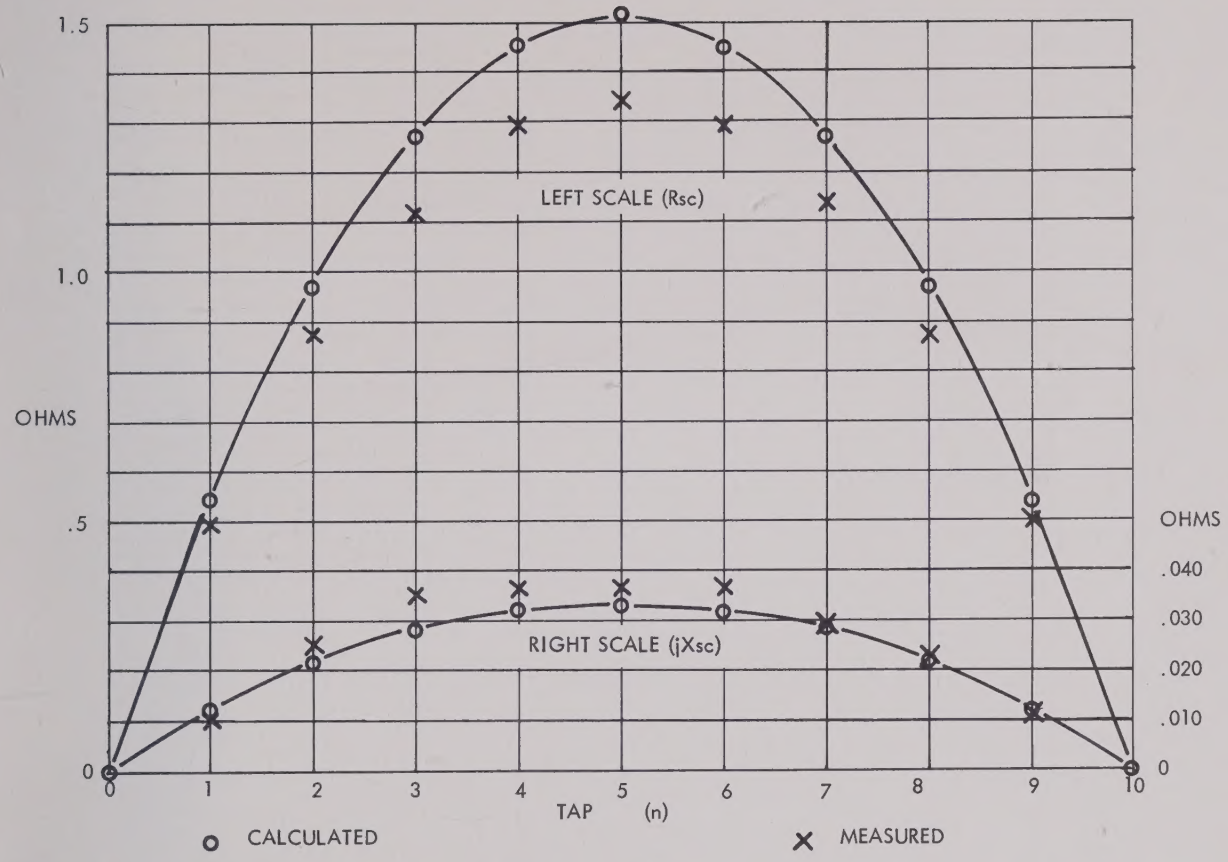


FIGURE. 5 OUTPUT IMPEDANCES [Zsc(n)] Vs TAP (n) at 400HZ

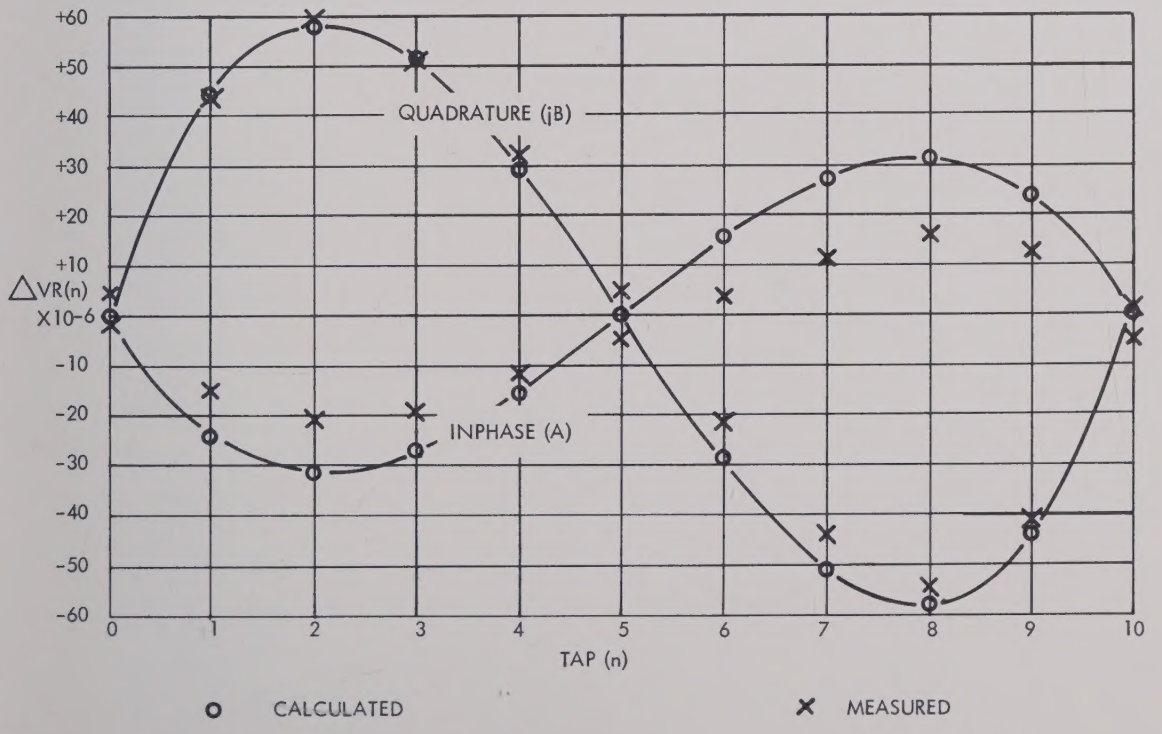


FIGURE. 6. RATIO ERRORS [△VR(n)] Vs. TAP (n) at 10KHZ